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BUSINESS MATHEMATICS

A TEXTBOOK

By

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PREFACE

The course in applied mathematics outlined in this book is much more advanced and thorough than the usual course in commercial arithmetic. The attempt has been made to construct a practical course which will contain all the essential mathematical knowledge required in a business career, either as employee, manager, or employer.

The fact that the field has been covered in this text both more intensively and more comprehensively than it has yet been covered in other texts, and the added fact that the material gathered together has stood the test of six years' experience in the teaching of large and varied classes, seem sufficient warrant for its publication.

The work is adapted not only for use in the classroom but also as a reference manual for those actively engaged in business life. Thus it will be found a practical guide for young employees who wish through private study to master the fundamental mathematics involved in "running a business." The tabulations, forms, illustrative examples, charts, logarithmic applications, and simple rules, are all applicable to the financial and other mathematical problems which business presents. Lack of knowledge of this side of a business, or inability to work out its mathematics, often results in haphazard guessing where accurate and careful calculations are required.

The material has been submitted to the criticism of many prominent business men and specialists in the commercial field, from whom valuable suggestions and criticisms have

been received. Many business publications have been drawn upon for illustrative material of various kinds: such books as "The U. S. Statistical Atlas," Finney and Brown's "Modern Business Arithmetic," Moore and Miner's "Concise Business Arithmetic," Van Tuyl's "Complete Business Arithmetic," Brinton's "Graphic Methods for Presenting Facts," Bowley's "Elementary Manual of Statistics," and Keister's "Corporation Accounting and Auditing," and such periodicals as *The Annalist*, issued by *The New York Times*, and *System*. There remains to acknowledge the helpful suggestions received from J. J. Hopkins, Ph.D., Principal of the Wm. L. Dickinson High School, and Frank Tibbetts, Head of the Commercial Department of the same school, as well as the critical reading of the manuscript by Hamlet P. Collins.

With whatever care a work of this character is prepared, it is realized that errors are liable to creep in undiscovered. The authors will be grateful to receive and glad to acknowledge any corrections or criticisms the reader may care to offer.

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BUSINESS MATHEMATICS

CHAPTER I

SALES AND PROFITS STATISTICS

1. Use of Comparative Records.—Every business is carried on for the purpose of selling something at a profit. The things sold may be the goods of the retailer, wholesaler, or manufacturer, or the services of an advertising concern, a bank, an insurance company, or public utility corporation. No matter what the kind of organization or enterprise, comparative figures of its sales and profits play an important part in its management.

The figures should be compiled regularly and tabulated to show increases or decreases covering corresponding periods of time. The tabulations may be made to show the trend of sales or profits by departments, lines of goods, or salesmen, or they may be worked out for the business as a whole. The computations involved seldom require more than the use of simple arithmetic and percentage calculations.

2. Comparative Percentage Figures.—In addition to showing sales and profits by quantities, it is advantageous to reduce these quantities to percentage figures, and show increases or decreases in this way. In finding percentages the proper adjustment should be made in the last decimal figure, up or down as the case requires.

Illustrative Example. The total sales for a given month are \$12,896.37, of which sales amounting to \$1,596.32 are credited to salesman A. Find, correct to two decimal places, the per cent of total sales made by A.

SOLUTION:

$$\$1,596.32 \div \$12,896.37 = .12378, \text{ or } 12.378\%$$

Adjusted to two decimals the last figure is 12.38%.

3. Tabulated Sales Records.—The tabulations most commonly used are those covering the daily, weekly, and monthly sales by departments or by salesmen. Columns may be ruled to show increases or decreases both by quantities and percentages or the cumulative figures to date, or the average figures reduced to percentages. The possible combinations are numerous and are determined by the kind of business.

4. Monthly Sales by Departments.—The figures in the tabulation shown below represent the monthly department sales totals for the year. The totals at the foot of each column give the annual sales of all departments.

Supply the missing totals at the foot of each column and in the last column.

MONTHLY RECORD OF SALES BY DEPARTMENTS

MONTHS	DEPT. 1	DEPT. 2	DEPT. 3	DEPT. 4	DEPT. 5	TOTAL
Jan.....	\$1,327.76	\$2,976.47	\$4,567.34	\$3,091.23	\$5,684.92	"
Feb.....	1,094.25	3,462.45	4,809.67	2,890.67	5,892.43	"
Mar.....	1,213.06	3,126.87	5,003.29	2,901.56	6,045.42	"
Apr.....	1,164.36	3,879.65	4,782.54	2,875.69	5,587.67	1821
May.....	1,086.79	2,580.56	4,347.83	2,784.35	5,469.57	"
June.....	987.57	2,784.66	4,476.21	2,783.52	5,472.31	"
July.....	975.64	2,564.43	4,357.81	2,569.58	5,216.49	"
Aug.....	976.56	2,376.65	4,235.68	2,467.92	5,127.65	15176.16
Sept.....	1,234.43	3,107.52	4,460.34	2,984.62	5,436.53	17263.44
Oct.....	1,321.26	3,245.63	4,532.25	3,012.56	5,542.32	17654.2
Nov.....	1,109.60	2,895.64	4,463.38	2,982.29	5,463.38	16914.7
Dec.....	1,437.87	3,256.76	4,987.56	3,248.90	5,873.24	18854.1
Total.....						

5. Daily Record of Sales by Departments.—The figures in the following tabulation give the daily sales in each department and are designed to show total daily and weekly sales both by departments and for the business as a whole. In the last line is shown the percentage of each day's sales to the grand total of sales, and in the last column the percentage of each department's weekly sales to the grand total of sales is shown.

Supply the required totals, and compute the percentages in each case as shown below.

COMPARATIVE DAILY RECORD OF SALES BY DEPARTMENTS

Week beginning.....

DEPT.	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTAL	% of GRAND TOTAL
I.	\$1,321.76	\$1,210.34	\$1,040.30	\$1,243.65	\$1,121.09	\$1,324.65		
II.	987.56	1,324.65	2,134.67	1,432.46	1,253.54	1,421.78		
III.	1,276.41	2,109.72	1,967.73	1,563.27	2,136.76	1,038.08		
IV.	2,107.63	2,371.10	1,986.78	2,083.52	2,376.82	2,171.19		
V.	987.64	1,097.47	897.74	1,107.27	985.71	1,207.24		
VI.	3,217.78	3,241.36	3,269.91	2,987.21	3,009.21	3,218.18		
Total..								
% of Grand Total								

6. Monthly Sales by Salesmen.—The tabulation below is designed to furnish the monthly total sales by salesman; the figures entered thereon are each salesman's weekly sales. This gives a comparison of salesmen's totals.

Compute the weekly and monthly totals, and compute the percentage of each salesman's sales to the total monthly sales correct to the nearest tenth per cent.

COMPARATIVE MONTHLY RECORD OF SALES

SALESMAN'S NUMBER	FIRST WEEK	SECOND WEEK	THIRD WEEK	FOURTH WEEK	TOTAL	% of GRAND TOTAL
1.....	\$350.65	\$567.87	\$436.65	\$654.43		
2.....	456.76	765.53	876.65	465.35		
3.....	387.57	476.53	587.76	745.36		
4.....	341.25	675.83	436.54	456.76		
5.....	324.43	546.67	478.35	765.28		
6.....	678.93	468.59	359.48	468.75		
7.....	426.47	578.64	658.47	536.28		
8.....	276.34	527.35	621.34	438.27		
9.....	576.27	364.14	713.26	465.27		
10.....	264.64	475.37	718.88	635.47		
Total.....						

7. Per Cent of Increase or Decrease.—The tabulation below gives the daily departmental sales figures and is designed to show the percentage of increase or decrease in each case.

Compute the percentages correct to one decimal place of per cent and indicate a decrease by an asterisk (*), or by red ink.

COMPARATIVE SALES FOR CORRESPONDING DAYS OF TWO YEARS

DEPT. NO.	SALES, WED. DEC. 1, 1910	SALES, WED. DEC. 3, 1920	INCREASE	DECREASE	% of IN. INCR.	% of DECR. DECR.
1.....	\$1,052.37	\$1,781.65			67.7	
2.....	1,342.54	1,254.46				67.7
3.....	1,254.32	1,324.56				
4.....	1,576.57	1,456.53				
5.....	2,564.34	2,657.62				
6.....	465.76	512.35				
7.....	1,467.43	1,547.25				
8.....	1,564.37	1,652.48				
9.....	1,231.12	1,235.65				
10.....	1,357.48	1,469.51				
Total....						

8. Per Cent of Average.—It is sometimes desirable to compare the weekly or monthly sales of a clerk or department, with the average weekly or monthly figures as the case may be.

In the following table compute the total and average sales and the per cent of the sales of each clerk to the average figures.

MONTHLY SALES OF A NUMBER OF CLERKS

CLERK'S NUMBER	SALES	PER CENT OF AVERAGE
350.....	\$1,256.43	
351.....	1,356.87	
352.....	1,124.34	
353.....	1,067.27	
354.....	987.56	
355.....	1,246.47	
356.....	1,456.32	
357.....	1,245.36	
358.....	1,034.75	
359.....	975.86	
360.....	1,326.52	
361.....	1,137.63	
362.....	1,364.37	
Total.....		
Average.....		

9. Sales Returned—Per Cent.—In some lines of business it is desirable to keep a close watch on goods returned. This can be effectively done by means of the percentage figures shown in the two following tables.

Compute the per cent of the departmental sales returned, to total sales, and the per cent of all sales returned, to total sales.

SALES AND RETURNED GOODS BY DEPARTMENTS

Year ending.....

DEPT.	SALES	RETURNED GOODS	NET SALES	% OF SALES RETURNED
1.....	\$ 25,431.76	\$ 768.63		
2.....	48,976.53	876.52		
3.....	76,432.56	1,097.57		
4.....	98,742.27	1,210.78		
5.....	67,834.62	895.68		
6.....	110,532.65	873.45		
Total.....				

10. Average Net Sales per Check.—Compute the total and net sales and the average net sales per check for each clerk.

INDIVIDUAL DAILY SALES SHEET

Section 13, Dry Goods

Date, May 11, 19—

CLERK'S NUMBER	GROSS SALES	RETURNED GOODS	NET SALES	CHECKS	AVERAGE NET SALES PER CHECK
121.....	\$312.67			121	
122.....	413.36	\$ 6.75		136	
123.....	215.23			97	
124.....	318.56	11.23		118	
125.....	456.78			124	
126.....	235.67	8.79		79	
127.....	102.46	1.78		81	
128.....	189.67	2.34		104	
129.....	213.53	3.21		115	
130.....	346.76	9.56		121	
Total.					

11. Tabulations for Other Comparative Purposes.—Tabulations similar to those given may be used to compare sales from month to month and from year to year, and also the month of one year with the same month in preceding years.

Show the monthly increase or decrease as the case may require and compute the percentage of increase or decrease of sales during the present year.

COMPARISON OF SALES BY CORRESPONDING MONTHS

Salesman, John Doe

MONTH	SALES LAST YEAR	SALES THIS YEAR	INCREASE	% OF INCREASE	DECREASE	% OF DECREASE
Jan.....	\$356.76	\$430.12	\$73.36	20.6		
Feb.....	456.87	515.60				
Mar.....	345.65	356.45				
Apr.....	450.10	390.50				
May.....	287.65	324.36				
June.....	231.90	245.87				
July.....	450.65	467.54				
Aug.....	346.75	356.52				
Sept.....	436.47	420.75				
Oct.....	567.35	580.67				
Nov.....	478.56	487.64				
Dec.....	564.32	545.53				

12. Cumulative Record of Sales.—By means of this form we may have not only a very complete record of each department's sales by any particular road or store salesman from month to month, but also a comparative record of the total sales for two or three or more months of any year, or a comparison of these totals for any previous year since this particular salesman has been connected with the business.

Compute the cumulative sales to the end of April and show the average at the foot of each column.

SALESMAN'S CUMULATIVE RECORD OF SALES BY DEPARTMENTS

Salesman, H. Williams

DEPT.	JANUARY	FEBRUARY	TOTAL 2 MONTHS	MARCH	TOTAL 3 MONTHS	APRIL	TOTAL 4 MONTHS
1.....	\$ 546.56	\$ 348.76		\$ 435.65		\$ 350.46	
2.....	768.56	756.46		675.87		763.54	
3.....	876.45	983.24		875.45		865.73	
4.....	1,134.76	1,234.58		1,346.43		1,265.87	
5.....	1,056.87	1,121.09		1,234.57		1,364.24	
Total.....							
Average...							

13. Computation of Loss or Gain.—In figuring the profits of a business, whether by departments or for the business as a whole, the deduction of the cost of the goods sold, from the goods sold or net sales, gives the gain for the period of time covered by the figures. To determine the cost of the goods sold it is necessary to deduct the cost of the goods unsold at the end of the month or year from the purchases made during the same period. Assuming that there are no goods on hand, i.e., no opening inventory at the beginning of the period, the computation would be as follows:

Goods sold (sales).....	\$70,000.00
Purchases.....	\$75,000.00
Cost of goods unsold (inventory end of year).....	10,000.00
Cost of goods sold.....	65,000.00
Gain.....	\$ 5,000.00
Gain %.....	$\$5,000 \div \$65,000 = 7.7\%$

WRITTEN EXERCISES

1. Goods sold during the year..... \$65,743.87
 Original cost of the goods..... 60,126.75
 Cost of goods unsold (inventory end)..... 2,234.76
 Find the gain and the gain per cent.
2. Goods sold during the year..... \$75,000.00
 Original cost of the goods..... 70,000.00
 Cost of goods unsold..... 2,564.85
 Find the gain and the gain per cent.
3. Sales..... \$80,000.00
 Purchases..... 90,000.00
 Inventory at end of year..... 5,000.00
 Find the loss and the loss per cent.

14. Accounting for Opening Inventories.—Assuming an opening inventory, then the gain for the period may be computed as shown below. Given the sales, inventories, and the cost of the purchases, this plan may be followed by any business to arrive at gain or loss.

Illustrative Example.

Sales.....	\$232,314.26
Opening inventory.....	\$ 36,756.65
Purchases of year.....	214,643.53
Total.....	<u>\$251,400.18</u>
Closing inventory.....	<u>75,654.78</u>
Difference.....	175,745.40
Gain.....	<u>\$ 56,568.86</u>

WRITTEN EXERCISES

1. A merchant had goods on hand January 1, 1920, \$46,756.87. During that year he purchased goods to the amount of \$314,567.89, and sold goods to the amount of \$298,654.65. His inventory at the end of that year showed goods on hand \$29,675.76.

Find his gain or loss for the year.

2. Inventory at the beginning of the year..... \$ 56,765.45
 Inventory at the end of the year..... 44,643.44
 Purchases during the year..... 347,124.96
 Sales during the year..... 357,649.85
 Find the gain or loss.
3. Purchases during the year..... \$250,669.25
 Sales during the year..... 276,040.34
 Beginning inventory..... 32,675.26
 End inventory..... 31,575.45
 Find the gain or loss.
 Find the gain or loss per cent.

15. Comparative Records and Statements.—Increases or decreases in profits of departments, salesmen, lines of goods, expenses, and sales, are sometimes compared as illustrated below. Percentages usually give the fairer comparison.

WRITTEN EXERCISES

1. Complete the records to show the required totals and percentages.

SALESMAN'S RECORD OF COMPARATIVE SALES BY DEPARTMENTS

Salesman, A. Jones

DEPT.	FIRST YEAR			SECOND YEAR			% OF INCREASE OR DECREASE
	Sales	Profits	%	Sales	Profits	%	
1.....	\$13,145.56	\$1,324.75		\$12,345.64	\$1,265.54		
2.....	3,256.53	578.97		3,567.56	585.74		
3.....	14,217.45	1,678.98		14,126.34	1,612.32		
4.....	4,352.36	412.36		4,215.76	398.67		
5.....	5,243.43	487.63		4,978.34	423.53		
6.....	14,689.57	1,265.42		14,357.26	1,196.57		
Total..							

2. Find the totals and percentages required in the following:

COMPARATIVE STATEMENT OF SALES, EARNINGS, AND EXPENSES FOR
THREE YEARS

	FIRST YEAR	SECOND YEAR	THIRD YEAR	TOTAL	AVERAGE
Income:					
Gross sales	\$896,437.65	\$1,062,792.80	\$963,416.50		
Less: Cost of goods sold	569,568.26	703,610.15	692,519.46		
Ratio to sales					
Gross profits on sales					
Ratio to sales					
Expenses:					
Selling expenses	\$129,562.75	\$ 146,927.64	\$119,816.75		
Ratio to sales					
Administrative and general expenses	76,837.56	87,167.35	72,963.10		
Ratio to sales					
Total expenses					
Ratio to sales					
Net profit on sales					
Ratio to sales					
Profit and loss charges.	\$ 16,283.16	\$ 10,341.62	\$ 7,189.42		
Ratio to sales					
Net Profit					
Ratio to sales					

3. How would you check Exercise 2? Check it.

The following is another form of profit and loss statement in which each item is a certain percentage of sales, as shown at the right.

CHECK: It will be noticed that the total of the figures in italics subtracted from the total of the other figures should equal 100.

PROFIT AND LOSS STATEMENT

Sales—Less Returned Goods.....	\$1,080,416.70	
Less Discount and Freight Allowance.....	75,215.63	6.96%
	<u>\$1,005,201.07</u>	
Cost of Goods Sold.....	\$ 749,153.80	
Add Inventory, January 1...	2,663,774.56	
	<u>\$3,412,928.36</u>	
Less Inventory, December 31.	2,711,355.62	64.94%
	<u>701,572.74</u>	
		<u>\$303,628.33</u>
<i>Add:</i>		
Purchase Cash Discount.....	\$ 6,864.07	.64%
Interest Received.....	\$ 101.44	.01%
Miscellaneous Income.....	268.97	.02%
		<u>7,234.48</u>
Gross Profit.....	\$310,862.81	
<i>Less:</i>		
Selling Expense.....	\$ 153,215.58	14.18%
Administration Expense.....	\$ 31,171.73	2.89%
Taxes—State and Federal...	11,530.89	1.07%
Interest Paid.....	4,028.30	.37%
	<u>46,730.92</u>	
Reserve for Bad Debts.....	5,402.08	.50%
Miscellaneous Expense.....	686.17	.06%
	<u>206,034.75</u>	
Net Profit.....	<u>\$104,828.06</u>	<u>9.70%</u>
		<u>100%</u>

CHAPTER II

PROFITS BASED ON SALES

16. Methods of Marking-up Goods.—In the marking of goods bought for resale the percentage of profit may be added to the cost price of the goods, which is their invoice price plus freight and cartage; or the percentage of profit may be computed on the sales, which is their cost price plus the expense of carrying on the business. The business man who adds a certain per cent of profit to the cost price of the goods rarely knows how much profit he is actually making, because he rarely knows how much the overhead expense of carrying on the business is. If, for instance, his sales are \$50,000 and his expenses for the year are \$10,000, it is apparent that each \$1 worth of goods costs \$.20 to sell. Therefore, to make 20% clear profit on his goods he must first add 20% to the cost price to give him their gross cost price, and then the 20% profit required. Thus the sales represent the cost to buy and sell.

17. Overhead Expenses.—Overhead expenses are those incurred in doing business—such as rent, taxes, salaries, light and heat, insurance, telephone, advertising, postage, depreciation, etc. These expenses must be taken into consideration when marking-up goods. Overhead expenses usually have a fairly constant ratio to gross sales, and from experience the merchant determines what this ratio is. This percentage plus the percentage of profit decided upon is deducted from 100%—representing the selling price—to determine the

percentage which the cost of goods plus freight bears to the selling price. The cost and freight figured in dollars divided by this percentage will give the selling price in dollars. This allows for the computation of the overhead and gain as a percentage of the sales.

Illustrative Example. An article is invoiced at \$23; overhead charges amount to 12% of sales and the gain is 8% of sales; freight is \$1. Find the selling price.

$$\begin{aligned}\text{SOLUTION: } 12\% + 8\% &= 20\% \\ 100\% - 20\% &= 80\% \\ \$23 + \$1 &= \$24, \text{ the cost} \\ \$24 \div 80\% &= \$30, \text{ the selling price}\end{aligned}$$

$$\begin{aligned}\text{CHECK: } 20\% \text{ of } \$30 &= \$6, \text{ the overhead charges and gain} \\ \$30 - \$6 &= \$24, \text{ the cost}\end{aligned}$$

$$\text{Overhead \% on Sales} + \text{Net Profit \% on Sales} = \text{Gross Profit \% on Sales}$$

$$100\% \text{ or Selling Price \%} - \text{Gross Profit \%} = \text{Cost of Sales \%}$$

$$\text{Invoice Price} + \text{Freight} = \text{Cost}$$

$$\text{Cost} \div \text{Cost of Sales \%} = \text{Selling Price}$$

ORAL EXERCISES

1. If an article costing \$50 is sold for \$60, what is the gain per cent on the cost? What is the gain per cent on the selling price?
2. An article cost \$25 and was sold for \$30. What is the per cent of gain on the cost? On the selling price?
3. An article is sold for \$60. If the gain on the selling price is 25%, what is the gain? What is the cost? What is the gain per cent on the cost?
4. A man sold an article for \$8 and made $12\frac{1}{2}\%$ on the selling price. What was the per cent of profit on the cost?
5. A merchant sold an article for 15 cents, which cost him 12 cents. What per cent profit is that on the cost? On the selling price?
6. If I buy an article for \$1, and then mark it to sell for \$1.25, how much must it be reduced to sell at cost? What per cent is this on the marked price? On the cost?

7. If an article cost \$1, and you sell it for \$1.50, what percentage of profit do you make, minus overhead?

8. If overhead expense is 20% of sales, what will an article that cost \$1 and you sell for \$1.50, figure as profit?

WRITTEN EXERCISES

1. Complete the following form.

NUMBER	COST	MARKED PRICE	% OF REDUCTION TO PRODUCE COST
1.....	\$ 1.00	\$ 1.20	
2.....	.20	.25	
3.....	.60	.75	
4.....	.03	.05	
5.....	.09	.12	
6.....	10.00	15.00	
7.....	2.50	2.75	
8.....	.40	.50	
9.....	50.00	75.00	
10.....	3.50	7.00	

2. Find the gain per cent on the selling price of the following:

No.	COST	SELLING PRICE
1.....	\$ 30.00	\$ 40.00
2.....	50.00	70.00
3.....	6.00	8.00
4.....	12.00	15.00
5.....	16.00	20.00
6.....	108.00	120.00
7.....	130.00	150.00
8.....	.17	.20
9.....	.15	.20
10.....	.03	.05

3. A man sells goods for \$15,000. The overhead charges are 15% of sales, and the profits 10% of sales. The freight is \$125. Find the invoice price of the goods.

4. An automobile is invoiced at \$1,140. The freight charges are \$50. If we allow 15% for overhead and 15% for profit, what should be the selling price of the automobile?

5. An invoice of merchandise amounts to \$1,575.25. If the overhead charges are 20%, the gain 10%, and the freight \$125, find the selling price.

6. The invoice cost of a lady's coat is \$30, the overhead charges are 25%, the profit is 15%, and the freight is \$2. Find the selling price.

7. A merchant marked goods at 20% above the cost. Owing to the fact that these goods did not sell well he reduced them 20% and claimed that he was selling them out at cost. Find the amount of his error in per cent. If he had reduced them 25%, how much would he have lost?

8. If a man buys some articles and marks them so as to gain 25%, and then reduces them to cost in order to move them, what per cent must he reduce them on the marked price?

9. The invoice cost of an article was \$3.50. The freight is \$.25, the overhead charges are 20%, and the profit is computed at 15%. Find the selling price.

10. A retailer buys tables at \$40, which he marks to sell at a profit of 40% on the cost. On account of slow business he decides to retail them at 25% less on the marked price. At what price does he sell them? Does he gain or lose and how much? What per cent is this on the selling price?

11. An automobile is invoiced at \$3,000. The freight is \$50. If we allow 10% for overhead, and 20% for profit, what should be the selling price of the automobile?

12. Complete the following form:

No.	COST	GAIN	% ON SALES	% ON COST
1	\$ 5.00	\$ 2.00		
2	.16	.04		
3	.08	.02		
4	24.00	8.00		
5	500.00	250.00		

18. Adding Per Cent of Cost.—The tables given on the remaining pages of this chapter are “short-cuts” for quickly calculating profits and selling prices.

The following table shows the percentage of cost which must be added to effect a given percentage of profit on sales.

ADD % TO COST	TO MAKE % PROFIT ON SALES	ADD % TO COST	TO MAKE % PROFIT ON SALES
1	.99	26	20.63
2	1.96	27	21.26
3	2.91	28	21.88
4	3.85	29	22.48
5	4.76	30	23.08
6	5.66	31	23.66
7	6.54	32	24.24
8	7.41	33	24.81
9	8.27	33½	25.00
10	9.09	34	25.37
11	9.91	35	25.93
12	10.71	36	26.47
12½	11.11	37	27.01
13	11.50	37½	27.27
14	12.28	38	27.54
15	13.04	39	28.06
16	13.79	40	28.57
16⅔	14.29	41	29.08
17	14.53	42	29.58
18	15.25	43	30.07
19	15.97	44	30.56
20	16.67	45	31.03
21	17.36	46	31.51
22	18.03	47	31.97
23	18.70	48	32.43
24	19.35	49	32.88
25	20.00	50	33½

19. Computing Net Profits.—The following table shows the per cent of the net profit when the per cent of expense to sales and the per cent of mark-up are known.

If the cost of doing business figured on sales is represented in the top line of the table below, and the mark-up on the goods is one of the percentages shown in the first column to the left, the percentage of net profit will be found at the junction of the line and column.

TABLE FOR COMPUTING NET PROFIT

% OF MARK-UP ON THE GOODS %		COST OF DOING BUSINESS FIGURED ON SALES																
		10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%	
25	10		9	8	7	6	5	4	3	2	1	00	Loss 1 4	Loss 2 3	Loss 3 2	Loss 4 1	Loss 5 00	
33 $\frac{1}{3}$	15		14	13	12	11	10	9	8	7	6	5	7 $\frac{1}{2}$	6 $\frac{1}{2}$	5 $\frac{1}{2}$	4 $\frac{1}{2}$	3 $\frac{1}{2}$	
40	18 $\frac{1}{2}$		17 $\frac{1}{2}$	16 $\frac{1}{2}$	15 $\frac{1}{2}$	14 $\frac{1}{2}$	13 $\frac{1}{2}$	12 $\frac{1}{2}$	11 $\frac{1}{2}$	10 $\frac{1}{2}$	9 $\frac{1}{2}$	8 $\frac{1}{2}$	12 $\frac{1}{2}$	11 $\frac{1}{2}$	10 $\frac{1}{2}$	9 $\frac{1}{2}$	8 $\frac{1}{2}$	
50	23 $\frac{1}{3}$		22 $\frac{1}{3}$	21 $\frac{1}{3}$	20 $\frac{1}{3}$	19 $\frac{1}{3}$	18 $\frac{1}{3}$	17 $\frac{1}{3}$	16 $\frac{1}{3}$	15 $\frac{1}{3}$	14 $\frac{1}{3}$	13 $\frac{1}{3}$	17 $\frac{1}{3}$	16 $\frac{1}{3}$	15 $\frac{1}{3}$	14 $\frac{1}{3}$	12 $\frac{1}{3}$	
60	27 $\frac{1}{2}$		26 $\frac{1}{2}$	25 $\frac{1}{2}$	24 $\frac{1}{2}$	23 $\frac{1}{2}$	22 $\frac{1}{2}$	21 $\frac{1}{2}$	20 $\frac{1}{2}$	19 $\frac{1}{2}$	18 $\frac{1}{2}$	17 $\frac{1}{2}$	21 $\frac{1}{2}$	20 $\frac{1}{2}$	19 $\frac{1}{2}$	18 $\frac{1}{2}$	17 $\frac{1}{2}$	
75	32 $\frac{1}{2}$		31 $\frac{1}{2}$	30 $\frac{1}{2}$	29 $\frac{1}{2}$	28 $\frac{1}{2}$	27 $\frac{1}{2}$	26 $\frac{1}{2}$	25 $\frac{1}{2}$	24 $\frac{1}{2}$	23 $\frac{1}{2}$	22 $\frac{1}{2}$	21 $\frac{1}{2}$	20 $\frac{1}{2}$	19 $\frac{1}{2}$	18 $\frac{1}{2}$	17 $\frac{1}{2}$	
100	40		39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	

Illustrative Example. If your cost of doing business is 15% of your gross sales and you mark a line at 25% above cost, your net profit is 5% on sales, as shown in the tabulation. If your cost of doing business is 18% and you mark a line at 60% above cost, your net profit is 19½% on sales.

20. Finding the Selling Price.—The table below is used to find the selling price of an article after the desired net per cent of profit is added and when the cost to do business is known.

To find the selling price divide the cost (invoice price plus the freight) by the percentage found at the junction of the “desired net per cent profit” with the percentage of the “cost to do business.”

TABLE FOR FINDING THE SELLING PRICE OF ANY ARTICLE

COST TO DO BUSINESS	NET PER CENT PROFIT DESIRED																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	20	25	30	35	40	50	
15%	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	65	60	55	50	45	35	
16%	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	64	59	54	49	44	34	
17%	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	63	58	53	48	43	33	
18%	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67	62	57	52	47	42	32	
19%	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	61	56	51	46	41	31	
20%	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	60	55	50	45	40	30	
21%	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	59	54	49	44	39	29	
22%	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	58	53	48	43	38	28	
23%	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	57	52	47	42	37	27	
24%	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61	56	51	46	41	36	26	
25%	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	55	50	45	40	35	25	

Illustrative Example. Article cost..... \$60.00

Freight..... 1.20

\$61.20

You desire to make a net profit of..... 5%

It cost you to do business..... 19%

SOLUTION: Take the figures in column 5 on the line with 19, which is 76.

$$\$61.20 \div .76 = \$80.52, \text{ the selling price}$$

The percentage of cost of doing business and profit are figured on selling price

21. Per Cent of Profit on Sales.—The following table shows the per cents of profit made on the selling price when the per cents shown in the first column are added to the cost of the goods sold.

TABLE FOR COMPUTING PROFIT

5%	added to cost	=	4 $\frac{3}{4}$ %	profit on selling price				
8 $\frac{1}{2}$ %	"	"	"	"	=	7%	"	"
10%	"	"	"	"	=	9%	"	"
12 $\frac{1}{2}$ %	"	"	"	"	=	11 $\frac{1}{8}$ %	"	"
15%	"	"	"	"	=	13%	"	"
16%	"	"	"	"	=	13 $\frac{3}{4}$ %	"	"
17 $\frac{1}{2}$ %	"	"	"	"	=	15%	"	"
20%	"	"	"	"	=	16 $\frac{2}{3}$ %	"	"
25%	"	"	"	"	=	20%	"	"
30%	"	"	"	"	=	23%	"	"
33 $\frac{1}{3}$ %	"	"	"	"	=	25%	"	"
35%	"	"	"	"	=	26%	"	"
37 $\frac{1}{2}$ %	"	"	"	"	=	27 $\frac{1}{4}$ %	"	"
40%	"	"	"	"	=	28 $\frac{1}{2}$ %	"	"
45%	"	"	"	"	=	31%	"	"
50%	"	"	"	"	=	33 $\frac{1}{3}$ %	"	"
55%	"	"	"	"	=	35 $\frac{1}{2}$ %	"	"
60%	"	"	"	"	=	37 $\frac{1}{2}$ %	"	"
65%	"	"	"	"	=	39 $\frac{1}{2}$ %	"	"
66 $\frac{2}{3}$ %	"	"	"	"	=	40%	"	"
70%	"	"	"	"	=	41%	"	"
75%	"	"	"	"	=	42 $\frac{2}{3}$ %	"	"
80%	"	"	"	"	=	44 $\frac{1}{2}$ %	"	"
85%	"	"	"	"	=	46%	"	"
90%	"	"	"	"	=	47 $\frac{1}{2}$ %	"	"
100%	"	"	"	"	=	50%	"	"

WRITTEN EXERCISES

1. A man buys an article for \$125 and wishes to make 20% profit on sales. How much profit shall he compute on the cost?

2. A merchant buys an article for a certain sum of money and wishes to mark it to sell so that he can make $33\frac{1}{3}\%$ on the cost. What per cent profit is this equivalent to when computed on the selling price?

3. A merchant marks an article to sell for \$200, thereby making $33\frac{1}{3}\%$ on the cost. Find the cost. What is the equivalent per cent on the selling price?

4. If your cost of doing business is 12% of gross sales and you mark a line at 50% above cost, what is your net profit on sales?

5. If you mark a line 75% above cost and your cost of doing business is 16% of your gross sales, what is your net profit on sales?

6. If goods sold amount to \$1,000 and your cost of doing business is 15%, and you have marked the line at 40% above the cost, what is the net profit on the sales? What is the cost?

7. If you buy an article for \$12, and the freight is \$.75, and you desire to make a net profit of 10%, and it costs 18% to do business, what is the selling price?

8. If you desire to make a net profit of 8% on an article which costs \$.20, and it costs 16% to do business, what should be the selling price of that article?

9. If the selling price of an article is \$100, the net profit is 4%, cost of doing business is 16%, find the cost of that article.

10. If a man sells an article for \$100 and makes 26% on the selling price, what per cent does he make on the cost?

CHAPTER III

PAY-ROLL CALCULATIONS

22. Methods of Wage Payment.—Every manufacturing business and many mercantile concerns maintain a pay-roll department. The duty of this department is to keep the employees' time records or records of the quantity of work done, and, at the end of the week or other period, to compute the wages or salary earned by each employee. Where work is paid for by the hour, day, or week, or according to the number of pieces made, the computing of the pay-roll may be a simple arithmetical problem of multiplication. Where scales of wages vary with the efficiency of each workman, and where good work or quick work is rewarded by the payment of a premium or bonus in addition to the regular hourly or piece-rate wage, the problem may involve intricate fractional and percentage calculations. The method of computing the premium or bonus may be based upon the number of pieces produced within a given time or upon the amount of time saved in the performance of a given operation.

23. Efficiency Payment Systems.—The more complicated methods of wage payment are met with in the highly organized mechanical industries where the modern method of management known as "scientific management" is increasingly employed. Different types of industry often adopt different methods; and the efficiency expert who has

been responsible for the introduction of a special system of wage payment usually gives it his name to distinguish it from others.

24. Day-Rate System.—Wages which are based on time worked are usually computed at an hourly rate, with one and one-half times the regular rate for overtime and holiday work, and sometimes double time for holiday work and Sundays. The practice with respect to overtime wages varies with different manufacturing concerns. The time clock is generally used to record each employee's time, and the time cards serve as a basis for computing the wages of the employees.

WRITTEN EXERCISES

1. In the following section of a pay-roll, the regular working day is assumed to be 8 hr. If a man works more than 8 hr. on any single day, he is paid time and a half for overtime, although on some days he may work less than the number of hours in the standard day.

Make the required computations to show the wages due to each employee.

NAME	HOURS PER DAY						HOURS	TIME RATE	AMOUNT	OVERTIME	AMOUNT	TOTAL EARNED	AMOUNT ADVANCED	WAGES DUE
	M	T	W	T	F	S								
John Doe....	8	7	6	8	8	8	45	\$.40	\$18.00				\$5.00	\$13.00
H. Jones....	9	8	8	9	8	8		.40						
C. Tyne.....	8	7	8	6	9	8		.38						
Wm. Osborn	6	8	8	7	8	8		.36						
Wm. Malone	9	8	9	8	9	8		.40						
H. Orr.....	8	8	8	7	8	8		.34						
J. Warner...	9	8	9	8	8	8		.40						
H. Phelps...	8	8	8	6	7	7		.36						

2. Rule a pay-roll like the preceding model, enter the following data and find the amount of wages due to each employee. A full day is 8 hr., and time and a half is paid for overtime.

NO.	EMPLOYEE'S NAME	HOURLY RATE
1	Henry Jones.....	\$.34
2	Wm. Johnson.....	.30
3	Chas. Bell.....	.32
4	A. T. Wigham.....*	.35
5	G. R. Martin.....	.34

During the week ending Oct. 13, time cards were turned in by the foreman, showing the number of hours worked by each employee to be as follows:

Monday.....	1, 8 ¹ ; 2, 10; 3, 8; 4, 7; 5, 8
Tuesday.....	1, 7 ; 2, 8; 3, 8; 4, 9; 5, 6
Wednesday.....	1, 8 ; 2, 9; 3, 8; 4, 8; 5, 7
Thursday.....	1, 8 ; 2, 8; 3, 9; 4, 8; 5, 8
Friday.....	1, 8 ; 2, 8; 3, 9; 4, 7; 5, 8
Saturday.....	1, 6 ; 2, 8; 3, 8; 4, 8; 5, 6

The cashier has already advanced the following sums: to No. 1, \$3; No. 3, \$5; No. 5, \$2.50.

* Workman No. 1, 8 hr.

25. Piecework System.—Instead of basing the wage rate upon time, it may be based upon the quantity produced. The principle of all straight piecework systems is that the employee is likely to work harder and produce more if he is paid in proportion to his production than he would if paid by the day or hour. A pay-roll designed to record piecework wage payments is similar to the one used under the hourly rate system, excepting that no provision need be made for overtime, as overtime is generally paid for at the same rate as regular time.

WRITTEN EXERCISES

1. Complete the following pay-roll by determining the amount due to each employee,

PIECEWORK PAY-ROLL FOR WEEK ENDING DECEMBER 27, 192—

No.	NAME	OPERATION NO.	NUMBER PRODUCED						TOTAL	RATE	AMOUNT EARNED	AMOUNT ADVANCED	AMOUNT DUE
			M	T	W	T	F	S					
1	H. James . . .	214	15	16	14	17	15	12	89	\$.15	\$13.35	\$3.00	\$10.35
2	G. Stone	167	24	25	23	26	26	25		.13			
3	W. Rafferty .	315	31	34	32	30	34	32		.07			
4	F. Johnson . .	426	43	41	42	45	40	24		.05		2.00	
5	G. Williams .	274	27	28	26	25	27	20		.10			

26. The Differential Rate.—Under this plan a careful estimate is made of the number of pieces each employee can produce in a day, and each employee is expected to produce the standard number. If he produces less than the standard, he receives less per piece. If he produces more than the standard, he receives more than the standard rate.

This system is based upon the idea that the expense of heat, light, rent, power, etc., remains the same whether the employees produce a small amount of product or a large amount. By increasing the amount of the production, these expenses are distributed over a larger quantity of manufactured goods, and the cost of making each article is thereby decreased. The saving effected by increasing the output is divided between the owner of the factory and the workmen who by their skill and industry increase the output. On the other hand, the employees who by producing a small quantity increase the cost per article, are paid less.

27. Computing the Differential Rate.—The problem of determining the standard number of pieces which shall constitute a fair day's work is often a difficult matter. The employer is naturally desirous of basing the rate on the

amount of pieces produced or work done by his most efficient employees. The less efficient employees or those who are not temperamentally fast workers may be penalized if the rate is placed at so high a figure that they earn less than the standard rate. Usually a compromise is made by placing the rate at such a figure that all industrious workmen can easily earn the standard rate and only the inefficient or lazy fail to earn it.

Illustrative Example. If a manufacturer found by experience that the average workman in his factory could produce 10 articles per day, and that 35¢ per article could be paid for the work, he might then outline the following differential rate:

NO. OF PIECES PRODUCED	RATE PER PIECE
8	\$.33
9	.34
10 (standard number and rate)	.35
11	.36
12	.37
13	.38

If Williams produces 8 pieces, he receives $8 \times \$0.33 = \$2.64$.

If Hartman " 13 " " " $13 \times \$0.38 = \$4.94$.

WRITTEN EXERCISES

1. Work out a pay-roll blank showing each employee's production and wages, using the daily production record and table of rates given below.

No.	NAME	DAILY PRODUCTION					
		M	T	W	T	F	S
1	John Jones.....	16	17	16	17	14	15
2	Henry Edwards.....	17	17	18	18	19	17
3	Chas. Tyne.....	18	16	17	18	17	17
4	J. Williams.....	18	19	21	22	21	20
5	W. Smith.....	18	18	19	18	19	18

NO. OF PIECES	RATE	NO. OF PIECES	RATE	NO. OF PIECES	RATE
10	\$.22	15	\$.29	20	\$.37
11	.23	16	.31	21	.39
12	.24	17	.32	22	.41
13	.25	18 (standard)	.34	23	.43
14	.27	19	.35	24	.44
				25	.45

28. Task and Bonus Plan.—This plan of wage payment is sometimes termed the “Gantt Bonus Plan.” In principle it is based on an extra payment per hour in addition to the regular hourly wage rate when the worker exceeds the standard.

Illustrative Example. Assume that in an office where typewriter work is paid for at piece rates the standard task at which the bonus payment begins is 150 or more sq. in. an hour, and that the bonus begins with an extra payment of \$.012 per hr. Assume again that for every 2 in. above the standard task, the bonus increases by \$.0012 up to 158 in.; by \$.0016 from 160 to 168 in.; and by \$.002 for 170 in. and above.

The bonus payment per hr. in addition to the regular hourly rate would then work out as shown in the following table:

BONUS TABLE

SQ. IN. PER HR.	BONUS PER HR.
150	\$.0120
152	.0132
154	.0148
156	.0160
158	.0172
160	.0188
162	.0204
164	.0220
166	.0236
168	.0252
170	.0272
172	.0292
174	.0312

WRITTEN EXERCISES

1. By the aid of the above table, compute the total wages of the five operators below.

PRODUCTION AND SOLUTION OF THE EXAMPLE

OPERATOR No.	SQ. IN	HOURS	WAGE RATE	BONUS PER HOUR	TOTAL HOURLY RATE	TOTAL WAGES
1.....	2,160	51	\$.40			
2.....	4,150	41	.36			
3.....	3,510	43	.44			
4.....	3,968	35½	.50			
5.....	4,975	46½	.48			

2. Using the bonus table given above, compute the total earnings of the following six typewriter operators.

OPERATOR No.	SQ. IN.	HOURS	WAGE RATE	BONUS	TOTAL EARNINGS
1.....	5,948	46	\$.40		
2.....	7,902	45½	.42		
3.....	6,546	53½	.44		
4.....	11,190	47	.46		
5.....	8,704	54	.43		
6.....	11,118	46½	.45		

29. Halsey-Rowan Premium Rate.—Under this method the workman is paid a premium which is generally from one-half to one-third the value of the time saved. The time saved is computed by setting a standard task to be done within a certain time, and the difference between the standard time and the actual time (where the actual time is less than the standard time) constitutes the time saved on the task.

Illustrative Example. If the standard time for curling a dozen ostrich feathers is 3 hr., and a worker whose hourly rate is 50¢ does the task in 2 hr., he saves 1 hr., or 50¢. Therefore he receives \$1 for the 2 hr. work plus a premium of 25¢, this being half the value of the hour saved. He still has one hour left in which he can do other work for which he should receive at least 50¢, so that his total payment for 3 hr. work should be:

EXPLANATION:

2 hr. 1 doz. (actual time) at \$.50 per hr.....	\$1.00
$\frac{1}{2}$ hr. (premium)..... “ .50 “ “	.25
1 hr. saved..... “ .50 “ “	\$1.25
Total.....	\$1.75

30. Emerson Efficiency Wage-System. —By this plan an “efficiency” bonus is paid to those workers who maintain or exceed a given rate of production. The bonus added is a percentage of the wages earned at the regular hourly rate, and the percentage is calculated by dividing the standard time by the actual number of hours taken to do the work. If the worker reaches 70% efficiency, he receives a bonus of 1¢ on every dollar of wages; if 80%, 5¢ on the dollar; if 90% 10¢ on the dollar; if 100%, 25¢ on the dollar; and so on progressively.

The method of calculating efficiency is:

If A = actual time in hours
 S = standard time in hours
 E = efficiency per cent

Then $E = S/A$

Illustrative Example.

Standard time is 25.5 pieces per hr.
 “ “ “ 100 “ in 3.92 + hr.
 Wage rate is 50¢

If the worker makes 30 pieces, the standard time would be: $30 \times .0392 = 1.176$ hr.

If the actual time on the 30 pieces were 1.7 hr., the "efficiency per cent" would be: $1.176 \div 1.7 = 69.2\%$.

Referring to the bonus table given below, an efficiency of 69.2% corresponds approximately to a bonus per cent of $.7\%$.

The computation would then be as follows: 1.7 (hr.) $\times$ \$.50 (wage rate) $\times$ $.7\%$ (bonus factor) = \$.00595 (bonus earned.)

BONUS SCALE

EFFICIENCY %	BONUS %	EFFICIENCY %	BONUS %	EFFICIENCY %	BONUS %
69	.7	81	5.2	93	13.
70	1.	82	5.6	94	14.
71	1.4	83	6.	95	15.
72	1.7	84	6.5	96	16.
73	2.2	85	7.	97	17.
74	2.6	86	7.5	98	18.
75	3.	87	8.	99	19.
76	3.3	88	8.5	100	25.
77	3.7	89	9.	101	26.
78	4.	90	10.	102	27.
79	4.4	91	11.	103	35.
80	5.	92	12.	104	45.

Allow for 5% increase in bonus for each increase of 1% in efficiency above 104.

The "efficiency per cent" corresponding to pieces per hour and the bonus in cents is as follows:

PIECES PER HOUR	EFFICIENCY %	BONUS IN CENTS
50	70	1 per dollar of wage
60	80	5 " " " "
75	100	25 " " " "

WRITTEN EXERCISES

1. Complete the following tabulation from the following data:

Standard time is 100 pieces in 4 hr.

" " " 25 " per hr.

" " " 1 " in .04 hr.

Wage rate is 40¢ per hr.

NAME	NO. OF PIECES MADE	TIME		EFFI- CIENCY %	AMT. EARNED AT REG. RATE	BONUS EARNED	
		Actual	Standard			%	Amount
J. B. Roads..	50	1½	2				
H. Warner...	75	2½	3				
L. Smith....	100	3	4	1⅓	\$1.20	19	
A. R. Jones..	40	1½	1½				
W. L. Brown	80	2½	3½				

2. Tabulate the above-mentioned employees, with the same number of pieces of work and the same actual time, from the figures following:

Standard time is 100 pieces in 5 hr.

" " " 20 " per hr.

" " " 1 " in — hr.

Wage rate is 50¢ per hr.

31. Salaries on a Commission Basis.—The salaries of employees are usually fixed in their amount and no extra pay is given for overtime. An exception to this custom is sometimes made in the case of salesmen whose salaries are frequently paid on what is known as a commission basis; that is, a certain percentage of the sales of each employee is paid to him instead of a regular salary or a small salary may be paid and supplemented with a commission on sales made.

WRITTEN EXERCISE

1. The wages of a certain firm are determined on the following commission basis:

On goods sold in Department 1,	10%
" " " " "	2, 12%
" " " " "	3, 12%
" " " " "	4, 15%

SALESMAN	DEPT. 1	DEPT. 2	DEPT. 3	DEPT. 4
H. Jones.....	\$360.24	\$415.67	\$302.24	\$412.53
J. Platt.....	534.45	645.45	587.65	768.89
M. Porter.....	267.87	304.46	289.75	412.13
M. Miner.....	879.67	903.42	876.65	756.54

Plan a form for the above which will show:

- Each salesman's commission in each department.
- Total commissions paid to each salesman.
- Total commissions paid for each department.
- Total commissions paid to all salesmen.
- Total sales in each department
- Total sales of each salesman.
- Total sales in all departments.

32. Pay-Roll Slips.—It is the practice of many concerns to hand out the weekly wage in envelopes, each employee receiving an envelope containing the exact amount of his pay. In this case, a coin sheet is prepared as below.

WRITTEN EXERCISE

1. Find the total wages and the total of bills and coins required to pay the wages shown below. Check your work.

COIN SHEET

No.	NAME	WAGES	BILLS				\$.50	\$.25	\$.10	\$.05	\$.01
			\$10	\$5	\$2	\$1					
1	E. Jones.....	\$22.77			1		1	1			
2	H. Jester.....	30.67									
3	C. Betts.....	25.34									
4	J. Bartlett.....	28.78		1	1		1	1			
5	R. Dibble.....	19.60									
	Total.....	76.12		1	1	1	1		1	1	11

33. Currency Memorandum.—This is used to take to the bank to show the paying teller just what kinds of money are required and how many of each kind are required to pay the weekly pay-roll.

CHASE NATIONAL BANK
CURRENCY MEMORANDUM
New York, April 1, 192—
Depositor—John Doe

		DOLLARS	CENTS
5 Bills	\$1.....	5	
“	2.....		
4 “	5.....	20	
“	10.....		
“	20.....		
“	50.....		
“	100.....		
Coin: Pennies.....			6
5 “ Nickels.....			25
8 “ Dimes.....			80
6 “ Quarters.....	1	50	
4 “ Halves.....	2		
“ Gold.....			
Total.....		29	61

WRITTEN EXERCISE

1. Make a currency memorandum for the exercise in § 32.

CHAPTER IV

INTEREST

34. Nature of Interest.—Interest is payment for the use of money. The interest to be paid for a certain sum depends upon the time for which the sum is loaned as well as on the rate per cent charged. The principal is the sum loaned. The principal and interest, added together, are called the amount.

Legal interest is the interest determined according to a rate fixed by law. If no rate is stated in a business transaction, the legal rate is used. The exacting of interest at a higher rate than that fixed by law is known as usury, and is punishable by law.

Interest rates are limited to 6% in all states with the following exceptions: 5% in Illinois, Louisiana, and Michigan; 7% in California, Georgia, Idaho, Nebraska, South Carolina, and South Dakota; 8% in Alabama, Alaska, Colorado, Florida, Montana, Utah, and Wyoming; 12% in Nevada.

Lending money is generally beneficial to the lender because it allows him to receive a return for the money earned by him or others in the past. It also helps the borrower, because he is enabled to get a larger return from his work.

ORAL EXERCISES

1. If a man borrows \$20,000 to enable him to manufacture a new article, what is the sum called which he must pay for the money?

2. If he pays \$1,200 for the use of this money for 1 yr., what should he pay for its use for 2 yr.? For 3 yr.? For 3 yr. and 6 mo.?

3. Suppose at the end of 3 yr. and 6 mo. he sells out his business for \$100,000. If we deduct the sum borrowed and the interest on it, how much will he have left?

4. If a man borrows \$50,000 in one state at 6% and loans it in another state for 8%, what will he make in 1 yr.?

5. If a man pays you $3\frac{1}{2}\%$ for the use of \$1,000 of your money and loans it at 6%, what will he make on it?

35. Interest Problems in Business.—It is a simple matter to calculate interest for definite terms such as 1 yr., 6 mo., or 3 mo. In business, however, it is frequently necessary to calculate the interest earned or due on a given sum of money up to a certain date and this involves fractional calculations. This has led to the devising of methods and tables which simplify the calculation of interest for a given number of days. These methods are explained in this chapter.

36. Interest for Years and Months.—Such calculation may easily be made by the following method:

1. Express the time in years only.
2. Find the interest at the given rate on the given amount for 1 yr. and multiply this by the number of years.
3. Parts of a year may easily be reduced from months and days.

Illustrative Example. Find the interest on \$450 for 3 yr. and 3 mo at 5%.

SOLUTION: 3 yr. and 3 mo. = $3\frac{1}{4}$ yr.

\$450	principal
.05	rate

\$22.50 = interest for 1 yr. at 5%

$$\begin{array}{r} 3\frac{1}{4} \\ \hline 562\frac{1}{2} \\ 6750 \\ \hline \end{array}$$

\$73.12 $\frac{1}{2}$ or \$73.13 = interest for 3 yr. and 3 mo. at 5%

WRITTEN EXERCISES

Find the interest on:

1. \$240 for 2 yr. 6 mo. at 5%.
2. \$375 for 4 yr. 8 mo. at 6%.
3. \$325.16 for 9 mo. at 3%.
4. \$456.76 for 3 yr. 6 mo. at 7%.

37. Short Methods of Computing Interest.—The following principles and methods of computing interest are shortcuts for calculating interest when the rate is 6%.

To find the interest at 6% for:

6	da.	point off	3	places to the left in the principal.
60	"	"	2	" " " " " " "
600	"	"	1	" " " " " " "
6,000	"	"	no	" " " " " " "

WRITTEN EXERCISES

1. Find the total amount of interest at 6% on the following:

\$1,575 for 60 da.

1,575	"	90	"	Find int. for 60 da. divide by 2 and add.
1,242	"	42	"	" " " 6 " and multiply by 7.
1,356	"	40	"	" " " 60 " divide by 3 and subtract
1,476	"	65	"	" " " 60 " divide by 12 and add.
1,674	"	24	"	
873	"	80	"	
1,824	"	70	"	

2. Find the total amount of interest at 6% on:

\$1,948 for 45 da.

2,648 " 15 "

3,642 " 25 "

2,600 " 21 "

3,400 " 33 "

3,600 " 55 "

Find int. for 6 da., divide by 2, and multiply by 7.

Find 30 da. and 3 da. int. and add.

Find 60 da. and 5 da. int. and subtract.

3. Find the total amount of interest at 6% on:

\$1,673.00 for 72 da.

1,236.00 " 84 "

465.46 " 48 "

474.89 " 6 "

127.46 " 9 "

656.48 " 10 "

824.34 " 7 "

4. Find the total interest at 6% on:

\$ 4,568.47 for 18 da.

356.35 " 2 "

6,000.00 " 1 "

1,245.60 " 3 "

7,454.00 " 2 "

9,000.00 " 8 "

15,648.00 " 4 "

1,242.00 " 5 "

5. Find the total amount of interest at 6% on:

\$ 456.84 for 7 mo. 15 da. 7 mo. = 3×2 mo. + 1 mo.

1,264.00 " 1 yr. 2 mo. 10 da. 60 da. = 2 mo.

1,952.00 " 6 mo. 20 da.

1,000.00 " 1 yr. 6 mo. 12 da.

1,275.87 " 8 mo. 8 da.

Rule: Pointing off two places in any principal gives the interest at rates other than 6% as follows:

At 1% for 1 yr.

At 2% " 6 mo. or 180 da.

At 3% " 4 " " 120 "

At 4% " 3 " " 90 "

At $4\frac{1}{2}\%$ " " " 80 "

ORAL EXERCISES

1. At 5% for how many da.?
2. At $7\frac{1}{2}\%$ " " " "
3. At 8% " " " "
4. At 9% " " " "
5. At 10% " " " "
6. At 12% " " " "
7. At 15% " " " "
8. How is the number of days for the two-place point-off obtained?

WRITTEN EXERCISES

1. Find the total interest on the following:

\$1,256.75	for	1 yr.	at	1%
456.45	"	3 mo.	"	4%
876.48	"	6 "	"	2%
986.54	"	80 da.	"	$4\frac{1}{2}\%$
984.42	"	72 "	"	5%

2. Find the total amount of interest on the following:

\$ 800	for	40 da.	at	9%
800	"	20 "	"	9%
1,600	"	48 "	"	$7\frac{1}{2}\%$
1,600	"	8 "	"	$7\frac{1}{2}\%$
2,400	"	45 "	"	8%
3,000	"	15 "	"	8%
4,000	"	60 "	"	8%

Rule: To find the interest at:

$6\frac{1}{2}\%$,	add	$\frac{1}{2}$	to the interest at	6%.
$6\frac{3}{4}\%$,	"	$\frac{3}{8}$	" " " "	6%.
7%,	"	$\frac{1}{6}$	" " " "	6%.

ORAL EXERCISES

1. $7\frac{1}{2}\%$, add ? to the interest at 6%.
2. $7\frac{1}{2}\%$, " ? " " " " 6%.
3. 8%, " ? " " " " 6%.
4. 10%, divide 6% interest by 6, and multiply by what?
5. 12%, multiply 6% interest by?

Rule: To find the interest at:

$5\frac{1}{2}\%$, deduct $\frac{1}{2}$ from the 6% interest.

$5\frac{1}{4}\%$, “ $\frac{1}{8}$ “ “ 6% “

3% , divide 6% interest by 2.

Any rate, divide 6% interest by 6, and multiply by that rate.

ORAL EXERCISES

To find:

1. 5% , deduct ? from 6% interest.

2. $4\frac{1}{2}\%$, “ ? “ 6% “

3. $4\frac{1}{4}\%$, “ ? “ 6% “

4. 4% , “ ? “ 6% “

5. 2% , divide 6% interest by what ?

Illustrative Example. What is the interest on \$2,400 for 60 da. at 5% ? at 4% ? at 7% ?

SOLUTION:

$$\begin{array}{r} \$24 = 60 \text{ da. int. at } 6\% \\ 4 = 60 \text{ “ “ “ } 1\% \\ \hline \end{array}$$

$$\$20 = 60 \text{ “ “ “ } 5\%$$

$$\begin{array}{r} \$24 = 60 \text{ da. int. at } 6\% \\ 8 = 60 \text{ “ “ “ } 2\% \\ \hline \end{array}$$

$$\$16 = 60 \text{ “ “ “ } 4\%$$

$$\begin{array}{r} \$24 = 60 \text{ da. int. at } 6\% \\ 4 = 60 \text{ “ “ “ } 1\% \\ \hline \end{array}$$

$$\$28 = 60 \text{ “ “ “ } 7\%$$

WRITTEN EXERCISES

1. Find the total interest on the following:

\$1,225.00	for	6	da.	at	7%
1,175.00	“	12	“	“	5%
1,456.00	“	3	“	“	4%
1,524.00	“	15	“	“	8%
1,200.00	“	30	“	“	9%
1,800.00	“	60	“	“	$4\frac{1}{2}\%$
3,624.00	“	60	“	“	$7\frac{1}{2}\%$
1,464.76	“	24	“	“	3%
4,468.74	“	36	“	“	1%

Rule: Interchanging principal and time. To find the interest on \$600 for 53 da. at 6%, change this to finding the interest on \$53 for 600 da., at 6%.

WRITTEN EXERCISE

1. Find the total interest on:

\$ 600	for	25 da.	at	6%
60	"	17 "	"	6%
200	"	13 "	"	6%
1,500	"	115 "	"	6%
1,200	"	67 "	"	6%
1,000	"	123 "	"	6%
660	"	16 "	"	6%

Rule: To find accurate interest.

First: Find ordinary interest by above principles.

Second: Deduct $\frac{1}{3}$ of the ordinary interest from itself.

Example. Find the accurate interest on \$7,300 at 6% for 60 da.

SOLUTION:

$$\$73 = 60 \text{ da. int. at } 6\% \text{ (ordinary interest)}$$

$$1 = \$73 \div 73$$

$$\$72 = 60 \text{ da. int. " } 6\% \text{ (accurate interest)}$$

MISCELLANEOUS WRITTEN EXERCISES

1. Find the total interest on:

\$600	for	80 da.	at	$4\frac{1}{2}\%$
550	"	72 "	"	5%
780	"	45 "	"	8%
800	"	40 "	"	9%
720	"	60 "	"	$6\frac{1}{2}\%$

2. Find the total interest on:

\$ 640	for	60 da.	at	$6\frac{3}{4}\%$
1,000	"	60 "	"	$7\frac{1}{5}\%$
600	"	60 "	"	10%

\$1,440 for 60 da. at $5\frac{1}{2}\%$
 560 " 60 " " $5\frac{1}{4}\%$

3. Find the total interest on:

\$2,000 for 60 da. at $4\frac{4}{5}\%$
 800 " 60 " " $4\frac{1}{2}\%$
 800 " 60 " " $2\frac{1}{4}\%$
 800 " 60 " " 9%

4. If the ordinary interest on a certain sum of money for a certain time is \$250,098, at a given rate, what is the accurate interest for the same time and the same rate?

38. To Find the Time from Principal, Rate, and Interest.

—It is sometimes necessary to find the time at which a certain sum of money at a given rate will produce a certain amount of interest. This is computed by finding the interest on the given amount of money for the given rate for one year, and dividing the amount of interest required by the amount for 1 yr. The result will be the number of years and a decimal or fraction of a year.

Illustrative Example 1. If the interest on \$600 is \$108 at 6%, annually, find the time it was on interest.

SOLUTION:

$$\begin{aligned} \$600 \text{ at } 6\% \text{ for 1 yr.} &= \$36 \\ \$108 \div \$36 &= 3 \\ \therefore &3 \text{ yr.} \end{aligned}$$

Illustrative Example 2. If \$400 at 6% yields \$24 interest, find the time.

SOLUTION:

$$\begin{aligned} \$400 \text{ at } 6\% \text{ for 1 yr.} &= \$24 \\ \$24 \div \$24 &= 1 \\ \therefore &1 \text{ yr.} \end{aligned}$$

WRITTEN EXERCISES

1. If \$500 yields \$120 at 6%, find the time.
2. In what time will it take \$1,200 to produce \$210 at 5%?
3. How long will it take \$1,000 to produce \$260 at 6%?

39. To Find Interest Between Certain Dates.—It is often necessary to find the time between certain dates and then find the interest for that amount of time. This is accomplished by:

(a) Subtracting the dates as illustrated below.

Illustrative Example. Find the interest on \$400 at 6% from July 12, 1919, to Jan. 3, 1921.

SOLUTION:	yr.	mo.	da.
	1921	1	3
	1919	7	12
	1	5	21

EXPLANATION: Subtract. In doing so, borrow 1 mo., and add its equivalent 30 da. to the 3 da. in the minuend, then take 12 from 33 = 21. Next take 7 from 0, but in order to do this, we must borrow 1 yr. and add its equivalent 12 mo. to the 0, then take 7 from 12. Finally subtract 1919 from 1920. Then find the interest for the given time at the given rate on \$400.

(b) Finding the exact number of days from one date to the next date.

Illustrative Example. Find the number of days from Mar. 15, 1919, to June 26, 1919.

SOLUTION:	Left in March	16 da.
	April has	30 "
	May "	31 "
	Time in June	26 "
	Total	103 "

WRITTEN EXERCISES

1. Find the time between June 17, 1919, and Oct. 14, 1919.
2. Find the time from Apr. 1, 1919, to May 6, 1923.
3. Find the time from Dec. 1, 1919, to March 18, 1920.
4. Find the time from Jan. 1, 1920, to March 1, 1920.

(c) The table method of finding a date in the future.

Illustrative Example 1. Find the date 5 mo. from May 6, 1920; also 6 mo. from Oct. 20, 1920. Note that May is the 5th mo., adding 5, gives the 10th mo. (see the number on the left) or October; therefore Oct. 6, 1920, is the date. October is the 10th mo., $10 + 6 = 16$, and the 16th mo. (see the number on the right) is April; therefore the date is Apr. 20, 1921.

1	Jan.	-1	13
2	Feb.	+2	14
3	Mar.	-1	15
4	Apr.		16
5	May	-1	17
6	June		18
7	July	-1	19
8	Aug.	-1	20
9	Sept.		21
10	Oct.	-1	22
11	Nov.		23
12	Dec.	-1	24

Illustrative Example 2. Find the date 90 da. from Dec. 7, 1920. $12 + 3 = 15$ th mo. if in 3 mo., or Mar. 7, 1921; but note in the table that 2 da. (1 da. + 1 da.) must be subtracted for December and January, and 2 da. added for February. Therefore the date is Mar. 7, 1921.

WRITTEN EXERCISES

1. Find the date 6 mo. after Jan. 2, 1921.
2. " " " 60 da. " June 17, 1921.
3. " " " 90 " " Aug. 10, 1921.
4. " " " 45 " " Sept. 16, 1921.

40. Compound Interest.—Compound interest is the interest on the principal and on the unpaid interest after it becomes due. It is usually paid on the deposits in savings banks. Premiums in life insurance are determined by it, and it is also used in sinking funds.

The interest may be compounded quarterly, semi-annually, annually, or at any regular interval of time. Its collection is not permissible in some states.

Illustrative Example. Find the compound interest on \$1,000 for 3 yr. at 5%, compounded annually.

SOLUTION:

$$\begin{aligned}
 5\% \text{ of } \$1,000 &= \$50, \text{ 1st year's interest} \\
 \$1,000 + \$50 &= \$1,050, \text{ new principal 2nd yr.} \\
 5\% \text{ of } \$1,050 &= \$52.50, \text{ 2nd year's interest} \\
 \$1,050 + \$52.50 &= \$1,102.50, \text{ new principal 3rd yr.} \\
 5\% \text{ of } \$1,102.50 &= \$55.13, \text{ 3rd year's interest} \\
 \$1,102.50 + \$55.13 &= \$1,157.63, \text{ amount end of 3rd yr.} \\
 \$1,157.63 - \$1,000 &= \$157.63, \text{ compound interest}
 \end{aligned}$$

WRITTEN EXERCISES

1. Find the compound interest on \$1,200 for 4 yr. at 6% compounded annually.

2. Find the compound interest on \$3,000 for 4 yr. at 4% compounded semiannually.

3. A boy has \$1,000 deposited in a savings bank for him on his 14th birthday. If the bank pays 4% compound interest, semiannually, what amount will he have on his 21st birthday if there are no other deposits or withdrawals?

4. Find the difference between the simple interest on \$1,000 for 6 yr. at 5%, and the compound interest on the same amount for the same time at the same rate if the interest is compounded semiannually. If compounded quarterly.

5. Find the compound interest on \$5,000 for 2 yr. at 4% compounded quarterly.

6. If \$200 is deposited in a savings bank which pays 4% compound interest compounded semiannually, on Jan. 1, 1921, what will it amount to July 1, 1924?

7. What is the compound amount on \$975 for 3 yr. at 5%, compounded semiannually?

41. Compound Interest Table.—If a person has much compound interest calculation work to do, he should resort to the table. It is much easier, simpler, and quicker than the method explained in § 40.

This table shows the amount of \$1 compounded annually at the different rates.

Years	3%	3½%	4%	4½%	5%	6%	Years
1	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	4
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	5
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	7
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	8
9	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	9
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	10
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.468534	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.512590	1.618695	1.731676	1.851945	1.979932	2.260904	14
15	1.557967	1.675349	1.800944	1.935282	2.078928	2.396558	15
16	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	16
17	1.652848	1.794676	1.947901	2.113377	2.292018	2.692773	17
18	1.702433	1.857489	2.025817	2.208479	2.406119	2.854339	18
19	1.753506	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.806111	1.989789	2.191123	2.411714	2.653298	3.207136	20
21	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564	21
22	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537	22
23	1.973587	2.206114	2.464716	2.752166	3.071524	3.819750	23
24	2.032794	2.283328	2.563304	2.876014	3.225100	4.048935	24
25	2.093778	2.363245	2.665836	3.005434	3.386355	4.291871	25

Illustrative Example. Find the compound interest on \$4,000 for 5 yr. at 6%.

SOLUTION: \$1 compounded annually at 6% for 5 yr. amounts to \$1.338226, as shown by the above table.

$$4000. \times \$1.338226 = \$5,352.90$$

$$\$5,352.90 - \$4,000 = \$1,352.90, \text{ compound interest}$$

NOTE: If the interest is compounded semiannually, take $\frac{1}{2}$ the rate for twice the time.

If the interest is compounded quarterly, take $\frac{1}{4}$ the rate for 4 times the time.

Illustrative Example. Find the compound interest on \$4,000 for 5 yr. at 6%, interest compounded semiannually.

SOLUTION:

$$\frac{1}{2} \text{ of } 6\% = 3\%.$$

$$2 \text{ times } 5 \text{ yr.} = 10 \text{ yr.}$$

The amount of \$1 compounded at 3% for 10 yr. is \$1.343916.

$$\$4,000 \times \$1.343916 = \$5,375.66$$

$$\$5,375.66 - \$4,000 = \$1,375.66, \text{ compound interest}$$

WRITTEN EXERCISES

1. Find the compound interest on \$3,500 for 6 yr. at 4%, compounded annually.
2. To what sum will \$2,000 amount in 9 yr. if invested at 6%, interest compounded semiannually?
3. What is the compound interest on a loan of \$500 at 12%, compounded quarterly for 5 yr.?
4. What sum must be invested at 4% compound interest to amount to \$800 in 10 yr. if the interest is compounded annually?
5. What sum must be deposited on Jan. 1, 1921, so that on Jan. 1, 1931, with interest at 5% compounded annually, the amount will be \$1,000?
6. What is the value of a 10-year endowment life insurance premium of \$100.60, if placed at 6% compound interest, compounded semiannually, at the end of the 10 yr.?
7. If the average daily number of passengers carried on the Interborough subways and elevated lines of New York was 1,011,053 in 1920, and the average increase per annum is 6%, how many passengers must be provided for 25 yr. later?

42. Annual Deposits at Compound Interest.—The following table is very useful when one has to find the amount of \$1 deposited annually at compound interest for any number of years up to 25 inclusive. It is perfectly obvious that such an example would be an endless task without a table of this nature. Its practical use will become very apparent with the written exercises which follow it.

This table shows the amount of \$1 deposited annually at compound interest for any number of years to 25 inclusive.

Years	2%	3%	4%	4½%	5%	6%
1	1.02	1.03	1.04	1.045	1.05	1.06
2	2.0604	2.0909	2.1216	2.137025	2.1525	2.1836
3	3.121608	3.183627	3.246464	3.278191	3.310125	3.374616
4	4.204040	4.309136	4.416323	4.470710	4.525631	4.637093
5	5.308121	5.468410	5.632975	5.716892	5.801913	5.975319
6	6.434283	6.662462	6.898294	7.019152	7.142008	7.393838
7	7.582969	7.892336	8.214226	8.380014	8.549109	8.897468
8	8.754628	9.159106	9.582795	9.802114	10.026564	10.491316
9	9.949721	10.463879	11.006107	11.288209	11.577893	12.180795
10	11.168715	11.807796	12.486351	12.841179	13.206787	13.971643
11	12.412090	13.192030	14.025805	14.464032	14.917127	15.869941
12	13.680332	14.617790	15.626838	16.159913	16.712983	17.882138
13	14.973938	16.086324	17.291911	17.932109	18.598632	20.015066
14	16.293417	17.598914	19.023588	19.784051	20.578564	22.275970
15	17.639285	19.156881	20.824531	21.719337	22.657492	24.672528
16	19.012071	20.761588	22.697512	23.741707	24.840366	27.212880
17	20.412312	22.441435	24.645113	25.855084	27.132385	29.905653
18	21.840559	24.116868	26.671229	28.063562	29.539004	32.759992
19	23.297370	25.870374	28.778079	30.371423	32.065954	35.785591
20	24.783317	27.676486	30.969202	32.783137	34.719252	38.992727
21	26.298984	29.536780	33.247970	35.303378	37.505214	42.392290
22	27.844963	31.452884	35.617889	37.937030	40.430475	45.995828
23	29.421862	33.426470	38.082604	40.689196	43.501999	49.815577
24	31.030300	35.459261	40.645908	43.565210	46.727099	53.864512
25	32.670906	37.553042	43.311745	46.570645	50.113454	58.156383

Illustrative Example. Find the amount of \$10 deposited annually for 10 yr. in a savings bank paying 4% compound interest.

SOLUTION: In the column headed 4%, and down opposite 10 yr., we find that \$1 under the stated conditions will amount to \$12.486351; then \$10 will amount to $10 \times \$12.486351$, or \$124.86351.

WRITTEN EXERCISES

1. A man 28 yr. of age has his life insured for \$2,000 by taking out a 20 yr. endowment policy, for which he pays annually \$49.95 per \$1,000. If at the expiration of the 20th yr. he receives the face value of the policy, find the gain to the insurance company if money is worth 4% compound interest to them. (See above table.)

2. If the insured in Exercise 1 had died at the age of 37, would the insurance company have gained or lost, and how much?

3. A young man starts a savings bank account on his 16th birthday by depositing \$30. If he deposits \$30 every 6 mo. thereafter until he is 25 yr. of age, what amount will he have to his credit, if the bank pays 4% interest compounded semiannually?

4. What amount of money deposited in a savings bank paying 4½% annually will amount to \$1,000 in 20 yr.?

43. Sinking Funds.—A sinking fund is a sum of money set aside at regular periods for the purpose of paying off an existing or anticipated indebtedness, or of replacing a value which will disappear by depreciation, exhaustion, or termination.

The payment of a public or a corporation debt and the replacing of certain public, corporate, or private values due to depreciation or other causes are often made easier by regularly investing a certain sum in some form of security. The interest and principal from these investments from year to year form a sinking fund, which, it is planned, shall accumulate to an amount needed to redeem the debt when it falls due, or replace the value when it disappears.

Illustrative Example. A corporation sets aside annually out of profits of the preceding year \$25,000 for 20 yr. If this amount is invested at 4½% compound interest, compounded annually, find the amount at the end of the 20th yr.

SOLUTION: Amount of \$1 deposited annually for 20 yr. at 4½% = \$32.783137.

Amount of \$25,000 deposited annually for 20 yr. = $25,000 \times \$32.783137 = \$819,578.40$.

Refer to above table.

WRITTEN EXERCISES

1. At the beginning of each year for 10 yr. a certain company set aside out of the profits of the previous year \$25,000 as a sinking fund. If this sum was invested at 4% compound interest, compounded annually, what did it amount to at the end of the 10th year?

2. Jan. 1, 1910, a certain city borrowed \$100,000 and agreed to pay the same on Jan. 1, 1920. What sum should have been invested on Jan. 1, 1910, and each succeeding year for 10 yr. in bonds paying 5% compound interest, compounded annually, in order to pay the loan when it became due?

3. What sum must a city set aside and invest annually to build a school building costing \$50,000 if it is to be paid for in 20 yr. and the city receives $4\frac{1}{2}\%$ on the money thus set aside?

4. What sum must a large printing company set aside to meet the cost of a printing press, through depreciation, in 15 yr., if it cost \$5,000, and the money is worth 4% compounded annually?

A.

CHAPTER V

DEPRECIATION

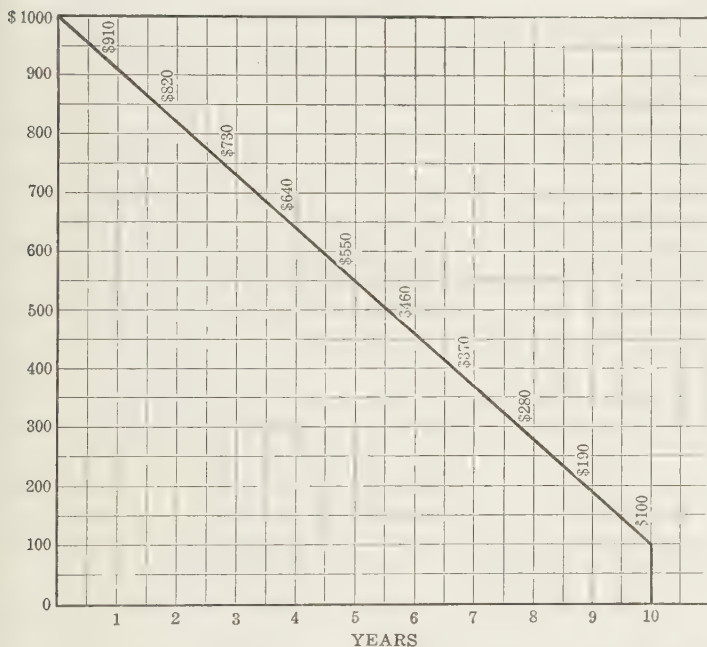
44. Nature of Depreciation.—Depreciation is the loss or expense incurred in business through decline in the value of property. While repairs may be made to prolong the usefulness of a building or a machine, sooner or later the time comes when the property is either worn out or it is good business economy to replace it.

A machine, for instance, costing \$2,400 is worn out in 10 yr., at the end of which time, when the machine is replaced, there will have been a loss of \$2,300 due to depreciation. Unless a portion of the depreciation is charged to profits as an annual expense, the entire \$2,300 loss will be charged against the profits of the last year. The practice in business is to spread this loss over the life of the property by charging off part of the loss to the operations of each year. These charges are called depreciation charges.

45. Methods of Computing Depreciation.—The following methods are those most commonly used to compute the depreciation charges:

1. The straight-line method.
2. A fixed rate, computed each year on the **original** value of the property.
3. A decreasing rate, computed on the **original** value of the property.
4. A fixed rate, computed on a **decreasing** value.

46. Straight-Line Method.—First, the probable life of the machine and the scrap value at the end of its life are determined. If it has been determined that 10 yr. is the



FORM 1. Depreciation Chart, showing Depreciated Value Computed According to the Straight-Line Method

life of the machine, then each year $\frac{1}{10}$ of the original cost of the machine less its scrap value is charged to factory expense account, and the depreciation reserve account is credited with the same amount. For example, if a machine cost \$1,000 and its probable life is 10 yr., and its scrap value is \$100, we take $\frac{1}{10} (\$1,000 - \$100) = \$90$, to be written off

each year. This is called the straight-line method or the fixed proportion method, because if the remainder values are plotted on the vertical lines (see Form 1) and the years on the horizontal line, then the remainder value is shown by the oblique straight line.

47. Fixed Rate Computed on Original Value.—This is a very simple method. The difference between the original value and the probable scrap value is first obtained. This difference is then divided by the number of years that the machine is estimated to last, and this result is called the depreciation per year. The depreciation per year is then divided by the original value of the machine, which gives the rate per cent of the original value to be charged off each year.

Illustrative Example. A printing press is purchased at a cost of \$8,000 and it is expected that this press can be used for 10 yr., when it will have a value of \$2,000. Therefore, during the 10 yr. of use, a depreciation of \$6,000 will occur. This is an annual depreciation of \$600. $\$600 \div \$8,000 = 7\frac{1}{2}\%$. Therefore $7\frac{1}{2}\%$ of the original value is charged off each year as an expense.

48. Decreasing Rate Computed on Original Value of Property.—It is sometimes preferred to charge the largest amount of depreciation the first year, gradually reducing it each year thereafter. This is done because a greater depreciation actually occurs during the first year than during any later year. For example, an automobile is “second-hand” after only a few months’ use, and the owner suffers a much greater loss from its use during the first year than he does during the second year. It will always depend upon the article as to what amount must be deducted each year.

49. Fixed Rate Computed on a Decreasing Value.—This method in a somewhat similar way as in § 48 results in a decreasing annual charge for depreciation. That is, the depreciation for the first year will amount to more than that for the second year, and that of the second year will be more than for the third year, etc.

Illustrative Example. Suppose the original value of the property is \$1,200 and the rate of depreciation is 10% a year, then depreciation under this method is computed as follows:

\$1,200.00	original value
.10	
\$120.00	depreciation first year
\$1,200.00	
120.00	
\$1,080.00	decreased or carrying value, beginning of second year
.10	
\$108.00	depreciation second year
\$1,080.00	
108.00	
\$972.00	decreased value, beginning of third year
.10	
\$97.20	depreciation third year, etc.

The **fixed rate** is obtained by somewhat more complicated calculations, which usually involve logarithms, and can readily be understood by anyone having a working knowledge of them. In general the fixed rate is found as shown in the following:

Illustrative Example. A printing firm purchased a printing press for \$5,000. It was estimated that it would last 10 yr. and have a scrap value of \$200. Find the annual rate of depreciation.

SOLUTION: The following equation is used:

$$r = 1 - \sqrt[n]{\frac{R}{V}}$$

where V = present value of the asset

R = residual value after n periods

n = number of periods

r = percentage of diminishing value to be deducted annually, or rate of depreciation

If we substitute the values of the problem we have:

$$\begin{aligned} r &= 1 - \sqrt[10]{\frac{\$ 200}{\$5,000}} \\ &= 1 - \sqrt[10]{.04} \\ &= 1 - .7221 + \\ &= .2779 - \end{aligned}$$

$\therefore 27.79 - \%$ will be the rate to be used on the decreasing value each year.

WRITTEN EXERCISES

1. Find the annual depreciation of a building worth \$15,560, if 4% is charged off each year.
2. How much is charged off annually for depreciation by a manufacturer who owns property which depreciates at the following rates?

PROPERTY	VALUE	DEPRECIATION RATE
Factory building.....	\$40,000	5 %
Machinery.....	4,800	7½ %
Tools.....	1,250	12½ %
Patents.....	5,000	6¼ %

3. The owner of a building estimates the annual depreciation as 3% of its cost. The building cost \$4,000. What is the amount of the annual depreciation?

The building is rented at \$40 per month. If the taxes, insurance, and other expenses amount to \$80 per year, what net income does the owner of this property receive on his investment after allowing for depreciation?

4. It is estimated that a machine costing \$2,220 can be sold at the end of 8 yr. for \$500. What per cent should be charged annually for depreciation?

5. Machinery in a factory cost \$24,000. Depreciation is computed as follows:

10% of the original value the 1st year

8% " " " " " 2d "

6% " " " " " 3d "

3% " " " " " 4th " and each year thereafter.

What was the amount of depreciation charged off each year for 5 yr.?

What was the inventory value of the machinery at the beginning of each year?

What was the inventory value of the machinery at the end of 12 yr.?

6. A flour mill was equipped with machinery costing \$60,000. Depreciation was computed at $12\frac{1}{2}\%$ of its cost the 1st yr., 8% of its cost the 2d yr., 5% the 3d yr., $2\frac{1}{2}\%$ the 4th yr., and 2% each year thereafter.

Find the amount of depreciation each year for 7 yr. Find the inventory value of the machinery at the beginning of each year.

7. Depreciation on certain property costing \$3,200, was computed at 8% of the decreased value for 4 yr. Find the annual depreciation and the decreased value each year.

8. A manufacturer was engaged in business for 10 yr. His machinery cost \$14,500, and he charged 6% depreciation annually on decreased values. Find the annual depreciation and the decreased value each year.

9. Machinery in a factory cost \$7,460, and depreciation was computed at 7% on decreased values. What was the depreciation during the 4th yr., and the inventory value at the end of the 4th yr.?

10. The cost of machinery was \$23,746, and depreciation was computed at $12\frac{1}{2}\%$ on decreasing annual values. What was the amount of the depreciation during the 6th yr., and the reduced value at the end of that year? The depreciation during this 6th yr. was what per cent of the original cost of the machinery?¹

¹The authors acknowledge their indebtedness to Finney and Brown, "Modern Business Arithmetic," for these problems and much of the other material in this chapter.

CHAPTER VI

INSURANCE

50. Necessity of Insurance.—Every business must take the precaution of insuring its premises and stock-in-trade against fire, its workmen against accident; and in many cases the life of a partner, a managing director, or an important officer, must also be insured to protect the business against the loss that his death might cause.

It is also considered wise for each employer or employee to insure himself against death or accident.

51. Kinds of Insurance.—There are very many kinds of insurance. The first four named below will be considered quite in detail in this work. Some kinds of insurance are:

- | | |
|-----------------------------------|---------------------------------|
| 1. Fire | 17. Transportation |
| 2. Life | 18. Keys (loss of) |
| 3. Fraternal | 19. Mail |
| 4. Accident | 20. Flood |
| 5. Liability | 21. Profit (loss of) |
| 6. Inspection | 22. Use and occupancy (loss of) |
| 7. Burglary | 23. War risk |
| 8. Plate glass | 24. Riot |
| 9. Steam boiler | 25. Damage |
| 10. Automatic sprinkler and claim | 26. Furniture |
| 11. Casualty | 27. Indemnity |
| 12. Automobile | 28. Musicians' fingers |
| 13. Live stock | 29. Earthquake |
| 14. Marine | 30. Title to property |
| 15. Hail | 31. Express |
| 16. Cyclone | 32. Health |

52. Fire Insurance.—Fire insurance is guaranty of indemnity for loss or damage to property by fire. Such contracts usually cover losses by lightning, and sometimes loss caused by cyclones and tornadoes. Insurance companies are liable for loss or damage resulting from the use of water or chemicals used in extinguishing the fire, and from smoke.

The fire insurance policies of all the companies in the states of New York, New Jersey, Connecticut, and Pennsylvania, are uniform and contain the "New York standard" (80%) clause. This in part is as follows:

"This company shall not be liable for a larger proportion of any loss or damage to the property described herein than the sum hereby insured bears to 80% of the actual cash value of said property at the time such loss shall happen." This is easily understood with an example.

For instance, if a piece of property is valued at \$100,000 and is insured for \$60,000, and a fire and water loss is \$40,000, the amount paid by the insurance company would be as follows:

$$\begin{array}{r} 80\% \text{ of } \$100,000 = \$80,000 \\ \$60,000 \\ \hline \$80,000 \end{array} = \frac{3}{4}$$

$\frac{3}{4}$ of \$40,000 = \$30,000 (amount paid by the company on this loss)

It will be observed that the company pays much less than the actual loss, owing to this clause in the policy. It has been claimed that the companies use this clause to force manufacturers and other large owners of property to insure their property for what it is worth. It can be easily seen that a large plant composed of many detached buildings is not as liable to burn up completely as a loft building situated in the city; and consequently the owner of the latter is keen

to insure his building for more nearly what it is actually worth, while the owners of large manufacturing plants are more liable to take a chance and not insure for what they are actually worth. Consequently the insurance companies by the aid of the 80% clause are able to penalize the large plant. The insurance company would have had to pay a much larger amount had the owners insured the property for \$100,000 at the beginning, as shown in the following computation:

$$\begin{aligned} 80\% \text{ of } \$100,000 &= \$80,000 \\ \frac{\$100,000}{\$80,000} &= 1\frac{1}{4}, \text{ or } 125\% \\ 1\frac{1}{4} \text{ of } \$40,000 &= \$50,000 \end{aligned}$$

Of course, it is hardly conceivable that the insurance company would have insured the plant for \$100,000 at the beginning; but they might have insured it for \$80,000 or \$90,000, and then the amount received for the loss would have been much larger than it was.

These policies also contain a "waiver" clause which is: "In case of loss, if the value of the property described herein does not exceed \$2,500 the 80% average clause shall be waived." Some states have passed laws which require the policy to state definitely the amount of loss for which the company is liable. By this policy the company is compelled to pay the actual loss not exceeding the face of the policy. In some states the policy contains a coinsurance clause, which specifies that only such a part of the loss will be paid as the face of the policy bears to the value of the property insured. If more than one company insures the same property, each company pays only its pro rata share of any loss on the property.

An **open policy** covers goods in storage and elsewhere. The amount varies as the quantity of goods is increased or decreased. When goods are received they are recorded. The premium charged is based upon the annual rate. If the goods are returned within 1 yr., the company returns the unearned part of the premium.

54. How to Find the Premium.—This is obtained by finding a certain rate on a certain number of dollars, or by finding a certain per cent of the amount of the policy, or by finding a certain rate in cents on a certain number of dollars with a possible discount on the latter in some cases.

SOLUTION: \$20,000 = 200 hundreds of dollars
 $200 \times \$18 = \$36$, the annual premium

SOLUTION:

$1\frac{1}{4}\%$ of \$12,000	= \$150
5% of \$150	= \$7.50
\$150 - \$7.50	= \$142.50, premium

WRITTEN EXERCISES

1. Find the premium on each of the following policies:

No.	FACE OF POLICY	RATE OF INSURANCE	AMOUNT OF PREMIUM
1	\$15,300	\$.21 per \$100	
2	17,500	\$.35 " \$100	
3	9,500	$\frac{1}{4}\%$	
4	23,500	$1\frac{1}{2}\%$ less 10%	
5	65,000	\$.45 per \$100 less 10%	

55. To Find the Amount Paid by the Insurer.

Illustrative Example. If property valued at \$50,000 is insured for \$30,000 at $\frac{3}{5}\%$ per annum, and fire and water cause a loss of \$24,000, find the amount that would be paid by the insurance company

- Under an ordinary policy
- Under a coinsurance clause policy
- Under the New York standard (80%) average clause policy

SOLUTION:

- (a) $\$24,000$
- (b) $\frac{\$30,000}{\$50,000} = \frac{3}{5}$
 $\frac{3}{5}$ of \$24,000 = \$14,400
- (c) 80% of \$50,000 = \$40,000
 $\frac{\$30,000}{\$40,000} = \frac{3}{4}$
 $\frac{3}{4}$ of \$24,000 = \$18,000

ORAL EXERCISES

- What amount of the loss does the company pay in (a)?
- State the part of the loss paid in (b) as a fraction. Write the names in the numerator and the denominator.
- Same as Exercise 2 with (c) in the place of (b).

Illustrative Example. A stock of merchandise is insured in Company X for \$10,000, in Company Y for \$14,000, and in Company Z for \$16,000. If the damage is \$10,000, how much should each company pay?

SOLUTION:

$$\$10,000 + \$14,000 + \$16,000 = \$40,000 \text{ total amount of insurance}$$

$$\frac{\$10,000}{\$40,000} = \frac{1}{4}; \frac{1}{4} \text{ of } \$10,000 = \$2,500, \text{ paid by Co. X}$$

$$\frac{\$14,000}{\$40,000} = \frac{7}{20}; \frac{7}{20} \text{ of } \$10,000 = \$3,500, \text{ paid by Co. Y}$$

$$\frac{\$16,000}{\$40,000} = \frac{2}{5}; \frac{2}{5} \text{ of } \$10,000 = \$4,000, \text{ paid by Co. Z}$$

$$\text{CHECK: } \$2,500 + \$3,500 + \$4,000 = \$10,000$$

WRITTEN EXERCISES

1. If a house is valued at \$12,000 and is insured for $\frac{2}{3}$ of its value at $\frac{3}{4}\%$, and its contents are valued at \$5,000 and are insured for $\frac{3}{4}$ of their value at $\frac{3}{5}\%$, and fire causes a total loss of the building and a loss of \$2,000 on the contents, find how much the insurance company will pay.

- Under an ordinary policy
- Under a coinsurance clause policy
- Under a New York standard 80% clause policy

2. A store and its contents are insured in Company A for \$40,000 at 55¢ per \$100; in Company B for \$48,000 at $\frac{3}{4}\%$; and in Company C for \$50,000 at 60¢ per \$100. This property is damaged by fire and water to the amount of \$20,000.

- What will each company pay?
- What is each company's net loss if they have held the insurance for 8 yr. when money is worth 5%?

56. Standard Short-Rate Table.—This table is used for the purpose of computing premiums for terms less than 1 yr., or for the purpose of computing the amount of premium to be returned by the insurer (the insurance company) when the policy is canceled by the insured. It is used as follows:

Take the percentage opposite the number of days that risk is to run, on the premium for 1 yr. at the given rate, and this result will be the premium to be charged in case of short risks, or earned in case of cancellation.

1 da.	2% of annual premium	50 da.	28% of annual premium
2 " 4%	" " "	55 " 29%	" " "
3 " 5%	" " "	60 " 30%	" " "
4 " 6%	" " "	65 " 33%	" " "
5 " 7%	" " "	70 " 36%	" " "
6 " 8%	" " "	75 " 37%	" " "
7 " 9%	" " "	80 " 38%	" " "
8 " 9%	" " "	85 " 39%	" " "
9 " 10%	" " "	90 " 40%	" " "
10 " 10%	" " "	105 " 45%	" " "
11 " 11%	" " "	120 " 50%	" " "
12 " 12%	" " "	135 " 55%	" " "
13 " 13%	" " "	150 " 60%	" " "
14 " 13%	" " "	165 " 65%	" " "
15 " 14%	" " "	180 " 70%	" " "
16 " 14%	" " "	195 " 73%	" " "
17 " 15%	" " "	210 " 75%	" " "
18 " 16%	" " "	225 " 78%	" " "
19 " 16%	" " "	240 " 80%	" " "
20 " 17%	" " "	255 " 83%	" " "
25 " 19%	" " "	270 " 85%	" " "
30 " 20%	" " "	285 " 88%	" " "
35 " 23%	" " "	300 " 90%	" " "
40 " 26%	" " "	315 " 93%	" " "
45 " 27%	" " "	330 " 95%	" " "
		360 " 100%	" " "

Illustrative Example 1. Find the cost of insuring a stock of goods for \$30,000 for 4 mo. if the annual rate is $\frac{5}{6}\%$.

SOLUTION:

$$\frac{5}{6}\% \text{ or } 1\% \text{ of } \$30,000 = \$300$$

$$\frac{1}{6} \text{ of } \$300 = \$50$$

$$\$300 - \$50 = \$250, \text{ annual premium}$$

$$50\% \text{ of } \$250 = \$125, \text{ premium for 4 mo}$$

$$6) \$300$$

$$50$$

$$2) \$250$$

$$\$125$$

EXPLANATION: $\frac{5}{8}\%$ = $\frac{1}{6}$ of the value of 1% less than the value of 1% of the amount.

Illustrative Example 2. Merchandise valued at \$48,000 is insured for $\frac{5}{8}$ of its value for 1 yr. at 55¢ per \$100. How much of the premium should be returned if the policy is canceled at the expiration of 9 mo. (a) by the insured? (b) By the insurance company?

SOLUTION:

- (a) $\frac{5}{8}$ of \$48,000 = \$40,000
 $400 \times \$55 = \$220$, annual premium
 15% of \$220 = \$33, amount returned if insured cancels policy
- (b) 3 mo. : 12 mo. = \$ x : \$220

$$\frac{3 \times \$220}{12} = \$55$$
, amount returned if insurer cancels policy

WRITTEN EXERCISES

1. An insurance policy for \$15,000 at $\frac{3}{4}\%$ per annum was dated Jan., 1921. Six months later it was canceled by the insured. How much of the premium was returned?

2. June 1, 1920, I took out a policy on my furniture for \$1,800 at 45¢ per \$100 per annum. Feb. 10, 1921, I canceled the policy.

(a) How much of the premium should be returned to me?

(b) How much would have been returned to me had the company canceled the policy on that date?

3. Goods valued at \$10,000 are insured at 60¢ per \$100 per annum for 3 yr. The policy is canceled by the insurer at the end of 2 yr.

(a) How much premium should be returned to the insured?

(b) How much would have been returned in case the policy had been canceled by the insured.

4. Find the cost of insuring a stock of goods for \$14,000 for 7 mo. at 70¢ per \$100 per annum.

5. An open policy of insurance is issued on merchandise stored in a warehouse, the premium on which is to be 75¢ per \$100 per year. Goods which are withdrawn within 1 yr. are to be charged the short rate. Find the total premiums paid, and the total returns on the following:

RECEIPTS			WITHDRAWALS			PREMIUM	TOTAL RETURN
Date		Amount	Date		Amount		
Feb. 10, 1920	Furs	\$15,000	Dec. 7, 1920	Furs	\$14,500		
Mar. 15, 1920	Silk	9,800	Aug. 25, 1920	Silk	9,000		
May 5, 1920	Woolen	8,900	Nov. 5, 1920	Woolen	8,500		
June 8, 1920	Hosiery	7,600	Dec. 4, 1920	Hosiery	7,600		
Aug. 7, 1920	Gloves	9,500	Nov. 25, 1920	Gloves	9,500		

57. Life Insurance.—Life insurance is a contract by which a company, in consideration of payments made at stated intervals by an individual (or by a company for the individual), agrees to pay a certain sum of money to his heirs at his death, or to himself if he attains a certain age.

The contract is called a **policy** and the money paid by the individual a **premium**. Premiums are payable either weekly, monthly, quarterly, semiannually, or annually.

Many policies now contain the permanent disability clause which states that the company shall waive payment of all future premiums and pay 10% of the face of the policy annually during disability, or make some other similar provision.

58. Principal Kinds of Life Insurance Policies.—The principal kinds of policies are:

1. Ordinary life policy
2. Limited life policy
3. Endowment policy
4. Term policy
5. Life income policy
6. Joint life policy
7. Survivorship annuity

They differ in three important ways:

1. The number of premiums paid by the insured
2. The amount of each premium
3. The time when payment is made by the company

59. Comparison of Different Kinds of Policies.—

KIND OF POLICY	NUMBER OF YEARS PREMIUMS ARE PAID	TIME WHEN PAYMENT IS MADE BY THE COMPANY
Ordinary Life	During life of insured. This period may be shortened in some companies if the dividends are allowed to accumulate.	At death of insured.
Limited Life		
10-payment life	10 yr.	At death of insured.
20- " "	20 "	At death of insured.
Endowment Policy:		
20-yr. endowment	20 yr.	At death of insured payment made to beneficiary; or at expiration of 20 yr. payment made to insured if still living.
10-yr. endowment	10 "	At death of insured or at expiration of 10 yr.
20-payment 30-yr. endowment	20 "	At death of insured or at expiration of 30 yr.
Term Policy:		
20-yr.	20 yr.	At death of insured if he dies within 20 yr. If he lives beyond this term no payment is made

Other periods of time may be obtained in the last three above.

Life income policy provides an annual payment to the insured after a stated date.

60. Premiums and Premium Rates.—The amount of the premium is a certain amount on \$1,000 worth of insurance, and depends upon the age of the insured at the time of buying the policy, and on the kind of policy. The younger the person, the cheaper the cost of the insurance.

The premium rates per thousand for different kinds of participating policies at different ages are shown in the following table.

ANNUAL PREMIUM ON DIFFERENT KINDS OF INSURANCE PER \$1,000

AGE	ORDINARY LIFE	20-PAY. LIFE	10-PAY. LIFE	10-YR. ENDOWMENT	20-YR. ENDOWMENT
15	\$17.40	\$27.34	\$44.63	\$100.60	\$47.79
20	19.21	29.39	47.85	101.57	48.48
25	21.49	31.83	51.67	102.73	49.33
26	22.01	32.37	52.51	102.99	49.53
28	23.14	33.52	54.28	103.54	49.95
30	24.38	34.76	56.18	104.14	50.43
35	28.11	38.34	61.53	105.87	51.91
38	30.88	40.89	65.21	107.13	53.10
40	33.01	42.79	67.90	108.07	54.06

61. Computation of Premiums.—

WRITTEN EXERCISES

From the preceding table find the annual premium for the following:

1. An ordinary life policy for \$3,000, taken by a man 25 yr. of age.
2. A 20-yr. endowment policy for \$4,000 for a man 20 yr. of age.
3. A man 28 yr. of age took out a 10-yr. endowment policy for \$4,000, and died after making 6 payments. How much less would the combined premiums have been on an ordinary life policy?
4. A man on his 26th birthday took out a 20-payment life policy for \$2,000, and 4 yr. later he took out an ordinary life policy for \$4,000. He died after making 12 payments on his first policy. How much more did the beneficiary receive from the insurance company than the insured had paid premiums? (Dividends not to be considered.)

62. Dividend Payments on Policies.—Certain companies write policies which provide that a portion of the profits of the company shall be paid to the holders of the policies. These annual payments of the share of the profits are called **dividends**. The profits of a company naturally vary from year to year, so that no specific amount can be guaranteed the policyholder. The policyholder generally receives no dividend the first year.

Dividends are applied, at the option of the insured, as follows:

1. Paid to him in cash.
2. Applied to the payment of his premium.
3. Left with the company and allowed to accumulate at compound interest.
4. Left with the company to increase the amount of insurance carried.
5. Left with the company in order to decrease the number of payments of premiums.

WRITTEN EXERCISES

1. Find the amount paid in in premiums on each of the following policies. Find the difference between the net cost of each policy and the amount received by the insured. How much per year did the protection he gave his family cost him?
2. Find the net cost for each year and the total net cost.
3. If the holder of these policies had died Nov., 1896, which policy would have netted the family the better returns?
4. How much more would the family have received than if the holder had put the same amount of savings in a savings bank paying 4% compound interest, compounded semiannually? (See § 42.)
5. Give reasons for a man's carrying life insurance.

The following table shows the actual dividends that were applied upon two different kinds of policies.

ANNUAL CASH DIVIDENDS AND NET COST OF INSURANCE ON POLICIES
OF \$1,000—AGE 25

YEAR	20-PAY. LIFE ISSUED IN 1893 ANNUAL PREMIUM \$27.28	NET COST	20-YR. ENDOWMENT POLICY ISSUED IN 1893 ANNUAL PREMIUM \$16.82	NET COST
1894	\$2.78	\$24.50	\$2.79	\$44.63
1895	2.95		3.17	
1896	3.14		3.56	
1897	3.34		3.97	
1898	3.53		4.40	
1899	3.73		4.85	
1900	3.93		5.31	
1901	4.17		5.80	
1902	4.15		5.31	
1903	4.26		5.23	
1904	4.41		5.58	
1905	4.56		5.89	
1906	4.71		6.21	
1907	4.97		6.54	
1908	5.04		6.88	
1909	5.21		7.23	
1910	6.18		8.64	
1911	6.36		8.99	
1912	6.56		9.35	
1913	7.09		9.72	

63. Cash Surrender, Loan, and Paid-up Insurance.—The policies of most companies have certain privileges which the insured may take advantage of after the policies have been in force for 2 or 3 yr.

These privileges are as follows:

1. Borrowing money from the company.
2. Surrendering the policy for a cash payment.
3. Receiving a "paid-up" policy which states a fixed amount of insurance during the remainder of life without further payment of premiums.
4. Being insured for the face of the policy for a fixed number of years and months.

For example, after a 20-yr. endowment policy for \$1,000, taken at the age of 33 in a certain company, has been in force for 10 yr., the insured can:

1. Borrow \$402.41 from the company on the security of the policy.
2. Surrender the policy and receive \$402.41 in cash.
3. Stop paying premiums and be insured for the remainder of his life for \$532.
4. Stop the payment of premiums and be insured for \$1,000 for 10 yr. or receive \$460 in cash at the end of 20 yr. from the date of the policy.

64. Methods of Settlement.—Upon proof of the death of insured (or in the case of an endowment policy, at the expiration of the endowment period) the policy is to be paid by the company. The various ways of settlement are:

1. Payment of cash to the beneficiary.
2. Annual payment of interest during the life of the beneficiary, and payment of the face of the policy at the death of the beneficiary.
3. Payment of equal annual instalments for the number of years specified to the beneficiary.
4. Payment of equal annual instalments for a certain period (usually 20 yr.), and for as many years thereafter as the beneficiary shall live. The amount of each annual payment depends upon the age of the beneficiary at the death of the insured.
5. Payment to the survivor of the face of the policy or of an annuity.

65. Lapses.—If the premium is not paid when due, the policy lapses. Most companies allow (and many states make it a law that they shall allow) 30 da. of grace during which time the premium may be paid plus the interest on that premium for the overdue time. If the insured wishes to pay the premium after this 30 days' time, he must undergo another examination by a physician and if successful he may pay it plus the interest on that premium for the time overdue.

66. Fraternal Insurance.—This is an insurance offered by different fraternal organizations. The following statements are answers to a questionnaire sent to officers or those thoroughly acquainted with the financial obligations of each of these organizations.

The arabic numbers in each group refer to the same number of group I, for example, III (3) refers to the Jersey City Teacher's life insurance organization.

I. Names of the organizations.

1. Now and Then Association.
2. Name omitted.
3. Jersey City Teachers.
4. Name omitted.
5. " "
6. United States Immigration Service Beneficial Association.

II. Amount paid to dependents of members at the death of the member.

1. \$.25 from each member of the association.
2. \$100 (some claim \$50).
3. \$1 from each member of the association.
4. \$1,000 or \$2,000.
5. \$500 to \$3,000.
6. \$1 from each member in good standing (average \$500).

III. How assessment to meet number II is paid.

1. Assessments upon the death of member, 20 da. in which to pay before second notice, with additional fee, called a tax, is sent.
2. Weekly dues 13¢.
3. Assessment of \$1 due by the first of the following month for each member who has died during the month.
4. At age 27 cost 99¢ per month. Increase is large for older men.
5. Age 27-29, 75¢ a month; monthly, semiannually (2% off); annually (4% off).
6. Assessment at death of member.

IV. To whom is money payable? When payable?

1. To one designated by deceased. Immediately upon proof of death.
2. To wife, if living, and if not, then to the children if any survive; if no wife or children, then to the nearest of kin. At once upon proof of death.
3. To the nearest of kin. Immediately upon proof of death.
4. To person designated.
5. To dependent (usually a relative) designated.
6. To anyone designated by the member. As soon as proof of death is shown (by official certificate or personal view of the remains by an officer of the society).

V. Is there a legal prior claim which can be put on this money?

1. None except for unpaid dues to the association.
2. None.
3. None.
4. None.
5. None.
6. None.

VI. Any other information you think advisable.

1. Dues in the association take care of insurance and other expenses of the club house. When the sick benefit fund reaches a stated amount an assessment is levied on members.
2. No reply.
3. A good thing from the point of view that the money is paid over immediately at a time when the dependents may

need it very much, while in an "old-line" company it generally takes from 30 to 60 da. to prove death and get the insurance.

4. Proof of death is sufficient to get the money.
5. \$.10 a month to pay supreme officers, etc. Company pays \$60 for funeral expenses.
6. It affords quick relief at small cost, now averages about \$16 per year per \$1,000. The only salaries paid are \$50 per annum for the secretary and treasurer.

VII. Amount of sick benefits. Are they uniform, or do they differ for different illnesses or accidents, and may a member pay more and receive more, or are they the same for all?

1. Sick benefits of \$5 per week, not paid for 1 week's illness.
2. \$4 per wk. for 13 wk.
- \$3 " " " " "
- \$2 " " " " "
- \$1 " " " " "

Uniform.

3. None.
4. Local organization (lodge) \$2.50 dues quarterly. Lodge raises money for sick members.
5. \$.50 once in 3 mo. for home expenses. Sick and accident benefits cost \$.50 a month, pay \$6 per wk. for 12 wk. Also pay \$750 for loss of both eyes, or both legs or both arms.
6. Amount of payments the same for all, depending upon the number of members in good standing. No accident or sick benefits. An annual ball or social affair is held, which usually nets about \$200 which pays incidental expenses.

67. Accident Insurance.—**Accident insurance** is insurance which covers loss by accidents. **Accident and health insurance** is insurance which covers loss to the insured through accidents, loss of health, or both.

Some important data for the layman to know follow.

68. What Constitutes the Occupation?—The profession, business, trade, employment, or any vocation followed as a

means of livelihood constitutes the occupation. Should a person engage in work, for hire, in any other occupation, such shall constitute his occupation.

69. Greatest Hazard Determines the Classification.—

If the applicant has more than one occupation, all occupations must be named in the application. The one involving the greatest hazard determines the classification.

70. Age of Applicants.—Applications will not be accepted from persons under 18 nor over 65, but those who insured before the age of 65 are usually carried to the 66th birthday. Disability or health insurance is issued only to male persons between 18 and 59

71. Beneficiaries.—They must always be named in the application blank, and must be persons having an insurable interest in the life of the insured, as a wife, father, mother, brother, sister, or other relative, a dependent or a creditor. If the insured does not care to mention a beneficiary, he can state "my estate" or "my executors, administrators, or assigns," in which case the money goes into the estate and has the same legal meaning as cash in the bank.

72. Scope of Policies.—The policies usually cover accidents sustained while residing or traveling for business or pleasure in any part of the civilized globe; while discharging the usual duties pertaining to the occupation named in the policy; while pursuing any ordinary form of pleasure or recreation; and while engaged in athletic exercises usually indulged in by business and professional men.

73. Limit of Risk.—The maximum amount of death benefit and weekly indemnity most companies will carry on any one risk (exclusive of the double clause) is stated opposite each classification, and cannot be increased except by special authority from the home office.

74. Prohibited Risks.—Persons are not insurable who are blind, deaf, or compelled to use a crutch or cane; who are insane, demented, feeble-minded, or subject to fits; who have suffered from paralysis or are paralyzed; who are intemperate, reckless, or disreputable; who are suffering from any bodily injury; or have any deformity, disease, or infirmity.

75. Cripples.—A person who has lost a hand, or a foot, or the sight of an eye, but is otherwise an able-bodied and acceptable risk, and whose occupation is not classed as more hazardous than ordinary, may be insured at an advanced rate by applying to the home office; but one who has lost a leg above the knee, or who is obliged to use a crutch or cane, or who, having any of the aforesaid defects, is engaged in an occupation more hazardous than ordinary, will not be accepted.

76. Insurance of Women.—A woman will not be accepted for weekly indemnity unless engaged in a stated business or employment from which she derives a regular income on which she is dependent for support. If in receipt of any such income, she may be insured for death benefit and weekly indemnity (without doubling clause) at the rate named for her occupation, but such policy will not be issued for more than \$3,000 death benefit and \$15 weekly indem-

nity. Housewives, housekeepers, boarding-house keepers, and canvassers are not to be included in the above.

A woman who, by reason of her circumstances and position in life, is not liable to loss or suspension of income on account of a disabling injury, may be insured under an accidental death policy at regular rates, but in no case for more than a maximum amount of \$5,000.

Disability or health policies are not issued to women.

77. Overinsurance.—Agents must guard against overinsurance of an applicant. The weekly indemnity should not equal the actual money value of the insured's time or of his weekly salary.

78. Designations of Occupations.—

A	1	Select
B	2	Preferred
BS	2+	Extra preferred
C	3	Ordinary
D	4	Medium
DS	5	Special
E	6	Hazardous
F	7	Extra hazardous
X		Prohibited

79. Industrial Insurance.—This is the kind of life insurance in which small investments can be made. Premiums, instead of being paid in a large sum once, twice, or four times a year in advance, are deposited in small sums, 3¢, 5¢, 10¢, 15¢, 20¢ a week, and so on, in exchange for which the company gives a life insurance policy payable at death of the insured, or after a certain number of years. Industrial insurance furnishes a means of saving, a little at a time,

week by week. Every time a 5¢ premium is deposited, something is saved. Every man who works for wages can secure the insurance protection which his means afford, without making a great strain on his income.

The companies usually send agents to the home every week to collect the premiums. This is done for the convenience of the policyholder, but if for any reason the agent should fail to call, the money may be sent to the home office of the company. One company was started in 1866, and now has more than twenty million policies of this kind in force. Any member of the family over the age of 1 yr. and up to the age of 65 yr. next birthday, who is in good health, can obtain one of these policies.

Illustrative Example. Suppose a young man of 25 pays 10¢ a week to the company. He secures a policy upon which he has to pay the weekly premium each week and the company agrees to pay in case of his death the sum of \$180, provided he has paid all premiums for 6 mo. or more up to the time of his death. If his death should occur any time within the first 6 mo. and he has paid regularly up to that time, the company would pay one-half of the \$180, even if he died immediately after the delivery of the policy to him. Some companies do not require the payment of further premium on any industrial policy after the insured reaches 75 yr. of age. Industrial endowment, cash values, paid-up insurance, paid-up endowment, and automatic extended insurance are common in this kind of insurance.

CHAPTER VII

EXCHANGE

80. Domestic Exchange.—Exchange is the payment of a debt for goods bought, or for some other purpose without the sending of money. Domestic exchange is such payment between persons or corporations in the same country.

ORAL EXERCISES

1. State some objections to sending money through the mails even if the letter is registered.
2. State some objections to sending it by express.

81. Methods of Exchange.—The paying of debts without the transmission of real money is effected by:

1. Personal checks
2. Postal office money orders
3. Bank drafts
4. Express money orders
5. Telegraphic money orders
6. Commercial drafts

If a merchant sends his check to a manufacturer in Detroit, the latter will deposit it in his bank, and it will be credited to his account. The check is then returned through proper channels to the bank of the maker, and the merchant is charged with that amount on his account.

If A owes B, in another town, \$25, and A does not have a bank account, he may go to the post-office and buy a money

order for the amount payable to B. This he sends by mail to B, which pays the debt. Find out from your post-office who is liable in case the money order and letter are lost or destroyed.

If A so wishes, he can go to a bank and pay the cash for a bank draft, payable to B, for the amount of the debt and then send it to B. B can take it to his bank and get the cash or have that amount credited to his account.

Or A can go to the express office and pay cash and buy an express money order payable to B, and send this on to B.

Or A can go to a telegraph office, pay cash and buy a telegraphic money order payable to B, and can send this by telegraphic communication to B in his city. B then can get the money at the telegraph office in his city.

Or if A is a merchant to whom a debt is due from some person who lives in B's town, A may send B a commercial draft drawn on C for the amount that A owes B.

82. Postal Money Order.—This is a government order by a post-office in one place to a post-office in some other place to pay a stated amount to a specified person. In order to obtain a postal money order a person must fill out an application blank which states:

1. The name and address of the payee.
2. The amount to be paid.
3. The name and address of the one who buys the order.

The rates charged for postal money orders are:

For \$ 2.50 or less	\$.03	From \$30.01 to \$ 40.00	\$.15
From 2.51 to \$ 5.00	.05	" 40.01 " 50.00	.18
" 5.01 " 10.00	.08	" 50.01 " 60.00	.20
" 10.01 " 20.00	.10	" 60.01 " 75.00	.25
" 20.01 " 30.00	.12	" 75.01 " 100.00	.30

The largest amount for which a single postal money order may be issued is \$100. If larger sums are to be sent, one must purchase additional money orders. These orders are to be presented for payment at the post-office on which they are drawn, or at a bank. If they are not presented within 1 yr. or if they are lost, a duplicate may be obtained from Washington upon proper presentation of evidence of such loss.

WRITTEN EXERCISE

1. Find the cost of the following money orders:

- (a) \$ 17.25
- (b) 4.50
- (c) 35.00
- (d) 64.13
- (e) 125.00

83. Express Money Orders.—An express money order (Form 2) is quite similar to a postal money order. It is a written order by one express agent to another agent to pay a stated sum to a specified person. The largest amount of an express money order is \$50. If one desires to send more he must purchase additional orders. Express money orders may be indorsed and transferred in a manner similar to bank drafts and checks. Rates are the same as for postal money orders.

WRITTEN EXERCISES

1. Find the total fee for the transfer of \$125 by express money order.
2. What is the best way to buy express money orders for \$87.75, and what would be the total cost of the same?
3. A man having a bank account prefers to send a check of \$37.50 to pay a bill he owes, rather than express money order or postal money order. Why?

A-3230104
American Express Co.
 REMITTER'S RECEIPT
 KEEP IT
 AMOUNT OF ORDER 01

A-3230104
American Express Company
 PAY TO THE ORDER OF Mr. J. P. Morgan
 THE SUM OF One Hundred Dollars
 DOLLARS AND NO CENTS
 65 BROADWAY, N. Y.
 DEC 31 1919
 Date

IF THE ABOVE DESCRIBED MONEY ORDER IS LOST OR DESTROYED THE COMPANY WILL REISSUE THE ORDER TO THE PARTY WHOSE SIGNATURE AND PRESENTATION OF THIS RECEIPT AND EXECUTION OF THE COMPANY'S AGREEMENT FOR REFUND

Form 2. Express Money Order

84. Telegraphic Money Orders. — Sometimes it becomes necessary to send money immediately for some special purpose, say one of the following:

1. To banks to meet maturing obligations.
2. To fire and life insurance companies for premiums.
3. To travelers and traveling salesmen.
4. To students and pupils at schools, seminaries, colleges, etc.
5. To guarantee purchases.
6. To accompany bids for contracts.
7. For payment of bills.
8. For the purchase of railroad, steamship, and theater tickets.
9. For purchases of all kinds.
10. For holiday gifts and other remembrances.
11. For memorial occasions and anniversaries.
12. For payment of taxes and assessments, and for all other purposes requiring the quick remittance of money.

Illustrative Example. To any place where the 10-word telegram rate is 60¢ one can send \$50 and a 15-word message for \$1.13.

WRITTEN EXERCISES

1. What is the total charge for sending \$30 by telegraph to a place where the cost of a 10-word message is 60¢?
2. Find the cost of sending \$250 by telegraph to the same place.
3. A man finds that he is in a strange place and needs money from his firm at once. He telegraphs for \$75. The charge for a ten-word message between those places is \$.72. Find the cost, including the charge for the message.
4. A man pays a bill of \$65.50 by postal money order, another bill of \$23.65 by express money order, and another bill of \$115 by telegraph. If the 10-word rate is 42¢, find the total cost to him.

In cases like these money may be sent by the use of a telegraph money order. The rates for such orders are obtained in a table like the one following.

TABLE OF CHARGES FOR TELEGRAPH MONEY ORDERS*

FOR A TRANSFER OF	TO ANY PLACE WHERE THE 10-WORD TELEGRAM RATE IS							
	\$.30	\$.36	\$.42	\$.48	\$.60	\$.72	\$.90	\$ 1.20
\$ 25.00 or less	\$0.68	\$0.74	\$0.80	\$0.91	\$1.03	\$1.22	\$1.45	\$1.88
25.01 to \$ 50.00	.78	.84	.90	1.01	1.13	1.32	1.55	1.98
50.01 to 75.00	1.03	1.09	1.15	1.26	1.38	1.57	1.80	2.23
75.01 to 100.00	1.28	1.34	1.40	1.51	1.63	1.82	2.05	2.48
100.01 to 200.00	1.53	1.59	1.65	1.76	1.88	2.07	2.30	2.73
200.01 to 300.00	1.78	1.84	1.90	2.01	2.13	2.32	2.55	2.98
300.01 to 400.00	2.03	2.09	2.15	2.26	2.38	2.57	2.80	3.23
400.01 to 500.00	2.28	2.34	2.40	2.51	2.63	2.82	3.05	3.48
500.01 to 600.00	2.53	2.59	2.65	2.76	2.88	3.07	3.30	3.73
600.01 to 700.00	2.78	2.84	2.90	3.01	3.13	3.32	3.55	3.98
700.01 to 800.00	3.03	3.09	3.15	3.26	3.38	3.57	3.80	4.23
800.01 to 900.00	3.28	3.34	3.40	3.51	3.63	3.82	4.05	4.48
900.01 to 1,000.00	3.53	3.59	3.65	3.76	3.88	4.07	4.30	4.73
1,000.01 to 1,100.00	3.78	3.84	3.90	4.01	4.13	4.32	4.55	4.98
1,100.01 to 1,200.00	4.03	4.09	4.15	4.26	4.38	4.57	4.80	5.23
1,200.01 to 1,300.00	4.28	4.34	4.40	4.51	4.63	4.82	5.05	5.48
1,300.01 to 1,400.00	4.53	4.59	4.65	4.76	4.88	5.07	5.30	5.73
1,400.01 to 1,500.00	4.78	4.84	4.90	5.01	5.13	5.32	5.55	5.98
1,500.01 to 1,600.00	5.03	5.09	5.15	5.26	5.38	5.57	5.80	6.23
1,600.01 to 1,700.00	5.28	5.34	5.40	5.51	5.63	5.82	6.05	6.48
1,700.01 to 1,800.00	5.53	5.59	5.65	5.76	5.88	6.07	6.30	6.73
1,800.01 to 1,900.00	5.78	5.84	5.90	6.01	6.13	6.32	6.55	6.98
1,900.01 to 2,000.00	6.03	6.09	6.15	6.26	6.38	6.57	6.80	7.23
2,000.01 to 2,100.00	6.28	6.34	6.40	6.51	6.63	6.82	7.05	7.48
2,100.01 to 2,200.00	6.53	6.59	6.65	6.76	6.88	7.07	7.30	7.73
2,200.01 to 2,300.00	6.78	6.84	6.90	7.01	7.13	7.32	7.55	7.98
2,300.01 to 2,400.00	7.03	7.09	7.15	7.26	7.38	7.57	7.80	8.23
2,400.01 to 2,500.00	7.28	7.34	7.40	7.51	7.63	7.82	8.05	8.48
2,500.01 to 2,600.00	7.53	7.59	7.65	7.76	7.88	8.07	8.30	8.73
2,600.01 to 2,700.00	7.78	7.84	7.90	8.01	8.13	8.32	8.55	8.98
2,700.01 to 2,800.00	8.03	8.09	8.15	8.26	8.38	8.57	8.80	9.23
2,800.01 to 2,900.00	8.28	8.34	8.40	8.51	8.63	8.82	9.05	9.48
2,900.01 to 3,000.00	8.53	8.59	8.65	8.76	8.88	9.07	9.30	9.73

\$3,000.00 and up add \$.20 for each additional one hundred dollars or fraction thereof to the charges for \$3,000.

* Including 15-word message.

85. Commercial Drafts.—Commercial drafts are used as an effective means of collecting an account overdue. It is an order drawn by the party to whom the money is due, asking the debtor to pay a specified sum of money to the drawer or to another person stated in the draft.

86. Kinds of Commercial Drafts.—1. One in which the debtor is asked to pay the money to the drawer. This kind of a draft is collected through a bank.

2. One in which the debtor is asked to pay the money to a third party.

87. Business Use of Commercial Drafts.—Company A of Binghamton, N. Y., sells A. H. Jones of Albany a shipment

\$500.00	BINGHAMTON, N. Y., April 14, 1921
..... At sight.....	pay to the order of
..... First National Bank of Binghamton.....	
Five Hundred and 00/100.....	Dollars
Value received and charge to the account of	
To A. H. JONES,	A COMPANY
Albany, N. Y.	
No. 589	

Form 3. Sight Draft

of shoes worth \$500. Company A wishes to get a quick return of the cash, or a promise from A. H. Jones that the latter will pay for them on a specified date. In order to do this Company A uses what is known as a bill of lading together with the commercial draft. Company A receives from the Binghamton freight office where these goods are shipped, this bill of lading, which is a written statement showing that these goods have been received there for ship-

ment. A. H. Jones must receive this bill of lading and present it to the freight office in Albany before he can obtain the goods. Company A takes this bill of lading to the First National Bank in Binghamton and deposits it along with a draft similar to that shown in Form 3.

The First National Bank of Binghamton sends this bill of lading and the draft to, say, the Albany Second National Bank, which in turn takes the draft to A. H. Jones for payment. If the latter pays it (or accepts it, in case it is a time draft), he then receives the bill of lading and in turn can secure the shoes from the freight office. Should A. H. Jones refuse to pay the draft, then the Albany Second National Bank so informs the Binghamton bank. The latter then notifies Company A, who must then sell the shoes in some other manner.

In case it is a time draft and A. H. Jones accepts it, the draft is then sent back to Company A, and the latter can take it to the bank and have it discounted and the proceeds credited to their account in the bank, or Company A can ask the Albany bank to discount it and send them the proceeds. If a time draft is discounted, time is figured from the day it is discounted.

88. Advantages of the Commercial Draft.—1. By the above method the purchaser must pay for the goods before he receives them.

2. If an account is past due, and a draft is sent to be collected by the purchaser's bank, it is often very effective in securing the money.

3. Company A, if they have good credit in their home bank, can immediately increase their funds because the home bank will discount the draft at once for them.

4. A. H. Jones cannot send a check which is worthless and thus cause Company A much loss in money and trouble.
5. The purchaser cannot claim that he has not received the bill of the goods.
6. The purchaser cannot maintain that he has already sent a remittance in some other way.

WRITTEN EXERCISES

1. If a commercial draft of \$3,000 payable 90 da. after date were discounted 15 da. after date, and the rate of discount were 6%, and the cost of exchange were $\frac{1}{2}\%$, find the net proceeds of this draft.

2. Suppose that R. D. Ford & Co. of New York were to sell A. H. Williams of Chicago a bill of goods amounting to \$1,500 on Apr. 14. Suppose that they draw a 60-da. draft on A. H. Williams through the First National Bank of New York. If a Chicago bank should buy this draft at 6% discount and exchange were $\frac{1}{10}\%$, what are the proceeds for R. D. Ford & Co.?

3. On Apr. 14, you purchased an invoice of goods of \$100 from H. R. Brown & Co. of Albany, N. Y. Terms, 30-da. draft from date of sale, less 2%. On Apr. 16, you received by mail the draft dated Apr. 14, and due in 30 days. You accepted it and returned it to H. R. Brown & Co. When will you have to pay this draft?

89. Terms Used in Domestic Exchange.—The **maker** or **drawer** of a draft is the person who signs it.

The **drawee** is the one who is to pay it.

The **payee** is the one to whom the money is paid.

Par means that a draft is bought exactly for its face value.

Premium means that the buyer of the draft must pay more than its face value. Exchange is then said to be at a premium. This occurs when, say, the banks of St. Louis owe the banks of New York a large sum, because the banks of St. Louis must either pay for the transportation of the money to New York or pay interest upon it. If A lived in

St. Louis at that time he would have to pay a premium because his draft would increase the amount that the St. Louis banks owe New York banks.

Discount is a term used when the draft can be bought slightly below face value. This might occur if D in New York were to send a draft to A, in St. Louis (given in the above paragraph) at the same time, because it would lessen the balance which New York had against St. Louis.

Exchange on checks is a small sum usually charged by a bank for paying a check from another city, since the bank is put to some expense in sending the check back and collecting the money on it. The sums vary from 10¢ up, but usually are not more than 25¢.

A **sight draft** is one which must be paid immediately when it is presented.

A **time draft** is payable after a stated time. It is presented at once to the drawee; if he accepts it, he writes the word "Accepted" across its face and signs it as well as dating it. This shows that he promises to pay it, and really makes it a promissory note.

90. Bank Drafts.—A bank in a small town, for instance, will keep funds in some large city bank on which it can draw checks just as an individual can draw a check on his bank for some other person.

A **bank draft** (Form 4) is an order drawn by one bank against its deposits in another bank.

91. Usefulness of Bank Drafts.—A bank draft is issued by the bank while a check is given by a person or a firm. The cashing of the former is considered safer than that of the latter.

There is less expense in collecting a bank draft than a personal check because the former are drawn on banks in large cities while the latter are very often drawn on small banks in the country.

EXPLANATION: The cashier of the Franklin National Bank of Franklin is W. D. Ogden. This bank deposits its funds with the Chase National Bank of New York. When

FRANKLIN NATIONAL BANK		No. 1687
FRANKLIN, N. Y., May 1, 1921		
Pay to the order of.....	E. H. Williams.....	\$25 $\frac{3}{8}$ %
Twenty-five and $\frac{3}{8}$ %.....		Dollars
To CHASE NATIONAL BANK of New York	W. D. OGDEN, Cashier	

Form 4. Bank Draft

the latter bank pays the amount of this draft it will reduce the balance of the account of the Franklin National Bank for this amount.

E. H. Williams, who lives in Franklin, wants to pay this amount to T. H. Morse of Albany. He accordingly buys this draft to send to Morse. He could have had it made payable to Morse, but if it should reach Morse with no letter to explain it, Morse might not credit it to the right man. Therefore Williams has it made payable to himself and, after receiving it, indorses it about as follows: "Pay to the order of T. H. Morse," and then signs his (Williams) name to it. This assures him of the proper credit by Morse and also acts as a receipt if Morse afterwards receives it and uses it.

Mr. Morse receives it and has it cashed at his bank, the Albany National Bank. The Albany National Bank col-

lects it as follows: This bank has a New York City bank, the Chemical National Bank. It therefore sends the draft to the Chemical National Bank as a deposit. The Chemical National Bank sends the draft to the Clearing House, which in turn collects it from the Chase National Bank. The latter bank upon paying the draft charges the amount of the draft to the Franklin National Bank.

92. Cost of Bank Drafts.—The Franklin National Bank would make a small charge for the draft. This charge is called exchange. The exchange would be paid by the purchaser (Williams). The usual charge is $\frac{1}{4}\%$ with a minimum charge of 10¢.

The following rates are in cents for \$1,000 between these cities.

Chicago—New York.....	\$.05 discount
San Francisco—New York.....	.25 premium
Boston—New York.....	.05 premium
St. Louis—New York.....	.20 discount

Illustrative Example 1. At 20¢ premium find the cost of a \$3,000 draft on New York in San Francisco.

$$\begin{array}{lcl} \text{SOLUTION:} & 3 \times & \$.20 = \$.60, \text{ premium} \\ & \$3,000 + & \$.60 = \$3,000.60, \text{ cost} \end{array}$$

$$\text{EXPLANATION:} \quad \$3,000 = 3 \text{ thousands}$$

Illustrative Example 2. If exchange is selling at 25¢ discount, find the cost of a \$3,000 draft.

$$\begin{array}{lcl} \text{SOLUTION:} & 3 \times \$.25 = & \$.75, \text{ discount on } \$3,000 \\ & \$3,000 - & \$.75 = \$2,999.25 \end{array}$$

WRITTEN EXERCISES

Using the above rates find:

1. The cost of a draft on New York at Chicago for \$13,600.
2. The cost of a \$12,800 draft at San Francisco on New York.
3. The cost of \$6,500 draft on New York at Boston.

93. To Find the Proceeds of a Draft.—

Illustrative Example 1. Find the proceeds of a sight draft of \$3,500, if the collection and exchange is $\frac{1}{8}\%$.

$$\begin{aligned}\text{SOLUTION:} \quad & \frac{1}{8}\% \text{ of } \$3,500 = \$4.375 \\ & \$3,500 - \$4.375 = \$3,495.625\end{aligned}$$

Illustrative Example 2. Find the proceeds of a 60-da. commercial draft of \$4,000, if sold the day it was dated at $\frac{1}{4}\%$ discount, when money is worth 6%.

$$\begin{aligned}\text{SOLUTION:} \quad & \$40 = 60\text{-da interest.} \\ & \frac{1}{4}\% \text{ of } \$4,000 = \$10 \\ & \$40 + \$10 = \$50, \text{ total discount} \\ & \$4,000 - \$50 = \$3,950, \text{ proceeds}\end{aligned}$$

WRITTEN EXERCISES

Find the proceeds of the following sight drafts:

1. \$1,800 draft when collection and exchange is $\frac{1}{8}\%$.
2. \$8,600 draft when collection and exchange is $\frac{1}{10}\%$.
3. Find the proceeds of a 90-da. draft of \$1,250.75 if sold at $\frac{1}{4}\%$ discount, when money is worth 6%.
4. Find the proceeds of a draft for \$75,000 when collection and exchange is $\frac{1}{4}\%$.
5. If collection and exchange is $\frac{1}{8}\%$ what are the proceeds of a draft for \$8,750?
6. What are the proceeds of a sight draft for \$1,375, if collection and exchange is $\frac{1}{4}\%$.

94. Postal Money Orders, Bank Drafts and Trade Acceptances —(a) The following differences exist between a postal money order and a bank draft:

1. A postal money order must be presented to the post-office on which it is drawn, or to some bank which can present it to that post-office.
2. A bank draft can be cashed at any bank.

3. A postal money order is to be indorsed but once.
4. A bank draft may be indorsed any number of times.
5. A postal money order will not be cashed until the post master receives a notice of such order from the office which wrote the order.
6. A bank draft can be cashed as soon as it is presented.

(b) A trade acceptance is like an ordinary bill of exchange except that it has a written guarantee upon it that the indebtedness has originated on an exchange of merchandise.

The advantage of the trade acceptance is shown by the following example:

A firm in Boston buys from a firm in New York \$1,000 worth of goods. Simultaneously with the shipment the seller draws on the buyer a draft at 90 days from date or sight (according to the terms of sale) and mails it to the latter with the invoice and the bill of lading. The usual form of draft is used with this additional clause; "the obligation of the acceptor hereof arises out of the purchase of goods from the drawer."

The buyer accepts the draft by writing across the face of it, "Accepted Payable at Bank, Boston." He dates and signs this acceptance and returns the accepted draft to the seller in New York. The document is now a "trade acceptance," becoming due for payment in Boston 90 days from the date of the draft, if drawn "after date," but in 90 da. from date of acceptance, if drawn "90 days' sight."

If the seller requires the money represented by this acceptance he may take the accepted draft to his bank and the bank will purchase it provided the names appearing thereon seem satisfactory. The bank in turn may rediscount the acceptance with the federal reserve bank, as the obligation arises out of the purchase of goods, and thus falls under the provisions of the Federal Reserve Act. The market rate of discount is charged by the bank for its courtesy.¹

¹Savay, Norbert, "Principles of Foreign Trade." New York, Ronald Press, 1919.

Manufacturers, wholesalers and jobbers are urging the use of trade acceptances, because they do away with much book-
ing and collections. Retailers are opposing it because they
have to keep a close watch on their bank account to meet
the payment of these acceptances.

95. Foreign Exchange and Foreign Money.—Our trade with foreign countries compels us to change United States money into the value of different foreign money, as well as to change foreign money into our money. International debts are settled by means of bills of exchange, postal money orders, bankers' bills, commercial bills, and the sending of actual money. Foreign exchange also deals with travelers' checks, letters of credit, etc.

96. How to Find the Value of Foreign Coins.—Find the value of the foreign coin in United States money. Multiply this value by the number of coins.

Illustrative Example. Find the value of £1,000 English money in United States money.

$$\begin{aligned}\text{£1} &= \$4.8665 \\ 1,000 \times \$4.8665 &= \$4,866.50\end{aligned}$$

∴ Multiply the number of coins by the value of one coin.

97. To Change United States Money into Foreign Money.—

Illustrative Example. Find the value of \$1,000 United States money in pounds English money.

$$\$1,000 \div \$4.8665 = \text{£}205.486$$

∴ Divide the number of dollars by the value of the foreign coin.

VALUES OF FOREIGN COINS IN UNITED STATES MONEY

COUNTRY	LEGAL STANDARD	MONETARY UNIT	VALUE IN TERMS OF U. S. MONEY	REMARKS
Argentine Republic	gold	peso	\$0.9648	
Austria-Hungary..	gold	krone	.2026	Greatly depreciated
Canada.....	gold	dollar	1.00	
France.....	gold, silver	franc	.193	Exchange value \$.0698
Germany.....	gold	mark	.2382	Greatly depreciated
Great Britain.....	gold	pound sterling	4.8665	Exchange value \$.9125
Italy.....	gold, silver	lira	.193	Exchange value \$.0409
Japan.....	gold	yen	.4985	Exchange value \$.5545

98. Bills of Exchange.—Bills of exchange are drafts of a person or bank in one country on a person or a bank in another country. They are divided into three classes:

1. Bankers' bills (Form 5).
2. Commercial bills drawn by one merchant on another.
3. Documentary bills, which are drawn by one merchant upon another and have a bill of lading attached, together with an insurance policy covering the goods en route.

Bills of exchange are usually issued in duplicate, called the original and the duplicate. They are sent by different mails and the payment of one of them cancels the other. Sometimes the original is sent, and the duplicate is placed on file and sent later if needed.

99. Par of Exchange.—This is the actual value of the pure metal of the monetary unit of one country expressed in terms of the monetary unit of another country.

Exchange for _____	New York _____
_____ of this Second of Exchange (First Unpaid)	
pay to the order of IRVING NATIONAL BANK, NEW YORK	
Value received, and charge the same to account of	
To _____	

Form 5. Bankers' Bill of Exchange

Deliver Documents only upon	Acceptance
Payment	

Illustrative Example. One pound sterling (£1) contains 113.0016 gr. of fine gold. \$1 contains 23.22 gr. of fine gold. $113.0016 \div 23.22 = 4.8665$. Therefore, £1 = \$4.8665, which is called the **par of exchange** between the United States and England.

100. Rate of Exchange.—This is the market value in one country of a bill of exchange of another country.

The price paid for a bill of exchange is constantly fluctuating, due like other things to supply and demand. If the United States should owe Great Britain the same amount that Great Britain owes the United States, then exchange would be at **par**. If the United States should owe Great Britain more than Great Britain owes the United States, then exchange in the United States would be at a **premium** and in Great Britain it would be at a **discount**. If Americans are exporting much more than they import, they will have many bills of exchange in the form of documentary bills to sell the American banker, and supply will exceed demand, thus causing exchange to fall **below par**. On the other hand, if Great Britain is exporting to the United States much more than the United States is exporting to Great Britain, then bills in the United States will be scarce and sell at a **premium**.

101. Quotations of Rates of Exchange.—Exchange on Great Britain is usually quoted at the number of dollars to the pound sterling; 4.86 means that a pound bill on London will cost \$4.86.

Exchange on France, Belgium, etc., is quoted at the number of cents to the franc. Thus, exchange on France quoted at 19.02 means that 1 franc costs 19.02 ¢.

Exchange on Germany is quoted at the number of cents to 1 mark. Thus, 8.5 means that 8.5¢ will purchase 1 mark.

The following foreign exchange rates were quoted on the date indicated:

NORMAL RATES
OF EXCHANGE

FEB. 1, 1921

\$1.8665	London.....	\$3.83½
19.30¢	Paris.....	7.04¢
19.30¢	Belgium.....	7.39¢
40.20¢	Holland.....	33.85¢
19.30¢	Italy.....	3.64¢
19.30¢	Spain.....	13.94¢
19.30¢	Switzerland.....	15.95¢
23.83¢	Germany.....	1.58¢

Illustrative Example 1. How to find the cost of a banker's bill on London:

What is the cost of a £500 draft on London bought at \$3.83½?

SOLUTION:

$$\begin{array}{r}
 \$3.835, \quad \text{cost of 1 pound} \\
 \quad \quad 500, \text{ number of pounds} \\
 \hline
 \$1,917.50, \text{ cost of 500 pounds}
 \end{array}$$

Illustrative Example 2. How to find the cost of a draft on Paris:

What is the cost of a 200-franc draft on Paris, bought at 7.04?

SOLUTION:

$$\begin{array}{r}
 $.0704, \text{ cost of 1 franc} \\
 \quad \quad 200, \text{ number of francs} \\
 \hline
 $14.08, \quad \text{cost of 200 francs}
 \end{array}$$

Illustrative Example 3. How to find the cost of a draft on Berlin:

What is the cost of a 2,000-mark draft bought at 1.58?

SOLUTION:

$$\begin{array}{r}
 $.0158, \quad \text{cost of 1 mark} \\
 \quad \quad 2,000, \text{ number of marks} \\
 \hline
 $31.60, \quad \text{cost of 2,000 marks}
 \end{array}$$

WRITTEN EXERCISES

Using the foregoing quotations, find the cost of drafts of each of the following amounts, first at the normal rate of exchange, and second at the quoted prices:

- | | |
|---------------|---------------------------|
| 1. £200 | 5. 1,800 marks |
| 2. £1,500 | 6. 240 marks |
| 3. 500 francs | 7. 150 guilders (Holland) |
| 4. 600 marks | 8. 350 guilders |

9. John Doe of New York owes T. H. Jones of London, £300 8s 5d. He buys a foreign draft of this amount to pay his bill. Suppose that exchange on London is \$4.75, what is the cost of the draft?

102. Letter of Credit.—This is a circular letter (Form 6) issued by some bank or banker, introducing the holder and instructing the bank's correspondents in stated places of the world to pay the holder any amount up to the face of the letter.

The holder deposits with the bank cash or securities to the face amount of the letter of credit. The purchaser must sign this letter at the purchasing bank in order that he may be properly identified by their correspondents. He also writes other copies of his signature which the bank forwards to its correspondents.

When the holder requires money he presents the letter to some one of the banks specified (as a correspondent) together with a draft drawn by the holder for the amount required. If the signatures agree he is paid the sum asked for, and such sum is indorsed on the back of the letter by the paying bank. The bank making the last payment retains the letter of credit and returns it to the drawee. Banks usually charge a commission of 1% for issuing a letter of credit.

The National City Bank of New York.

Letter of Credit

No 00000

New York July 1 1921

Gentlemen

We beg leave to introduce to you

and to commend to your courtesy

Richard Doane Smith

a statement of whose signature appears on the accompanying list of correspondents

kindly provide **Mr. Smith** with such funds as may

be required up to an aggregate amount of **\$5,000.00**

Five thousand dollars

against his deposit with the National City Bank of New York

per order

We engage that such drafts negotiated by you before the first day of **August 1, 1921** will be duly honored

The amounts paid must be endorsed upon the Letter of Credit and the drafts must state that they are drawn under N. C. B. Letter of Credit No 00000 dated **July 1 1921**

Your charges of any are to be paid by the beneficiary

Yours respectfully

The National City Bank of New York.

In Witness

the Bank is authorized in the accompanying list of correspondents

When returned the Letter of Credit must be marked "redeemed" in post box

103. Travelers' Check.—This is a circular check (Form 7) which is made payable for a stated amount in the currency of the foreign countries named on the face of the check. They are usually in amounts of \$10, \$20, \$50, \$100, and \$200. A commission of $\frac{1}{2}\%$ is the customary charge.

104. Postal Money Orders.—The following rates prevail for foreign postal money orders, if payable in Austria, Belgium, Bolivia, Cape Colony, Costa Rica, Denmark, Egypt, Germany, Great Britain, Honduras, Hongkong, Hungary, Italy, Japan, Liberia, Luxemburg, New South Wales, New Zealand, Peru, Portugal, Queensland, Russia, Salvador, South Australia, Switzerland, Tasmania, The Transvaal, Uruguay, and Victoria.

FOR ORDERS		COST	FOR ORDERS		COST
From \$	.01 to \$ 2.50	\$.10	From \$30.01 to \$	40.00	\$.45
"	2.51 " 5.00	.15	"	40.01 " 50.00	.50
"	5.01 " 7.50	.20	"	50.01 " 60.00	.60
"	7.51 " 10.00	.25	"	60.01 " 70.00	.70
"	10.01 " 15.00	.30	"	70.01 " 80.00	.80
"	15.01 " 20.00	.35	"	80.01 " 90.00	.90
"	20.01 " 30.00	.40	"	90.01 " 100.00	1.00

If payable in any other foreign country.

FOR ORDERS		COST	FOR ORDERS		COST
From \$	.01 to \$10.00	\$.10	From \$70.01 to \$	60.00	\$.60
"	10.01 " 20.00	.20	"	60.01 " 70.00	.70
"	20.01 " 30.00	.30	"	70.01 " 80.00	.80
"	30.01 " 40.00	.40	"	80.01 " 90.00	.90
"	40.01 " 50.00	.50	"	90.01 " 100.00	1.00

WRITTEN EXERCISES

1. Find the cost of a postal money order for \$35 sent to a person in Canada.

SPECIMEN 10 8000000
THE NATIONAL CITY BANK OF NEW YORK
 THROUGH WHICH THE WORLD
SPECIMEN
 WHEN NOT NEGOTIATED
 MONEY ORDER IS UNPAID, PAYEE'S CERTIFICATION
 WHEN COUNTERSIGNED BY THE BANK IS IN THE CREDIT OF THE BANK AS BROKER
 THE NATIONAL CITY BANK OF NEW YORK
 SPECIMEN
 TO THE ORDER OF THE OWNER IS PAID

Form 7. Travelers' Check

2. What will it cost me to buy \$250 worth of orders for a man in Paris?
3. How much would it cost me to buy the following orders made payable to myself in the following countries:

AMOUNT OF ORDER	PAYABLE IN
\$25	London
50	Paris
25	Constantinople
50	Calcutta

4. A sends 200 francs to B in Switzerland. If 1 franc costs 19¢, find cost of postal order.

105. Use of Commercial Bills.—If A in England owes B, a merchant in the United States, and B wishes to collect, he may draw up a commercial bill and let his bank collect it in a manner similar to the collection of sight or time drafts in domestic exchange.

106. Immediate Payment by Bill of Lading.—Mr. Jones, a merchant in the United States, sends a bill of goods to Mr. Williams, who lives in London. Mr. Jones delivers the goods to the transportation company and receives a bill of lading. He also insures the goods against loss in transit and receives the certificate of insurance from the insurance company. Mr. Jones then draws a bill of exchange on Mr. Williams, and attaches the bill of lading and the insurance certificate to this bill of exchange. All these papers are indorsed to the order of the bank which buys the draft. Mr. Jones then receives pay for the goods he has shipped. If the goods are lost the bank is reimbursed by the insurance company. If the bill is uncollectible, the goods are taken over by the bank. The United States bank indorses these papers and sends them to a foreign bank, thereby receiving credit by the latter for the amount. The foreign bank then collects the bill.

MISCELLANEOUS EXERCISES

1. At 25¢ a word and 1% of the amount, find the cost of a 25-word cable money order from New York to Paris for 20,000 francs, if exchange is quoted at 7.04 (cents).

2. An exporter sold to a broker the following bills of exchange: £500 at 3.90; 1,250 francs at 7.18; and 12,400 marks at 1.58. Find the total net proceeds, if the broker charged $\frac{1}{8}\%$ for collection.

3. What would be the cost of a London draft for £220, exchange being quoted at 4.85?

4. A man sends his son \$200 to London. How much English money would the son receive if exchange is quoted at 4.8665?

5. A Frenchman presents to a New York bank a draft for 1,000 francs. What money should he receive if exchange is quoted at 7.21?

CHAPTER VIII

TAXES

107. Kinds of Tax.—Many businesses as well as many persons are required to pay some form of tax. A tax is money levied upon a person or property for the payment of the public expenses.

A **direct tax** is a tax levied on a person, his property, or his business. If it is upon his business it usually takes the form of a license fee, and if upon his person it is called a **poll tax**.

An **indirect tax** is a tax (called a **duty**) on imported goods or a tax (called an **internal revenue**) on tobacco products. The latter tax need not be paid on goods exported.

An **income tax** is a tax on the income of a person or a firm.

An **excess profits tax** is a tax upon the excess profits of a business.

The taxes are levied by officers called **assessors**, or by people called **income tax collectors**.

108. Purpose of Taxes.—The purpose of taxes is to meet the expenses of the government. These purposes may be classified somewhat as follows:

1. National taxes are to pay the army and navy, the salaries of the officers and employees, pensions, and for any other United States government expenses.
2. State taxes are to pay their officers and employees, to support their schools, universities, asylums, and to pay all state expenses.

3. County taxes are to pay for the cost of the roads, the salaries of employees, for charities, and for any other county expenses.
4. City taxes are to pay for fire and police protection, the salaries of employees, the support of the schools, and for any other city expenses.
5. Town taxes are to pay for their schools, the salaries of employees, and for any other town expenses.

109. Method of Assessing Taxes in a State.—The state legislature determines the amount of money to be spent. The amount of taxable property is usually determined by local officers called assessors. The total amount to be collected is then divided by the number of dollars of taxable property. Therefore the tax is a certain per cent of the property assessed.

The property is usually assessed at some part of its real value. This part will vary in different localities or states. In some states it is becoming the policy to assess for nearly the full value.

110. How to Find Amount of a Tax.—The amount of the tax is also determined by various methods. The following examples will perhaps explain the different methods in the clearest way.

Illustrative Example 1. When the rate is stated as a certain number of mills on the dollar:

Warner's property is assessed at \$4,000; the tax rate is 35 mills on the dollar. Find his tax.

SOLUTION:	\$4,000 assessed valuation
	.035 tax rate
	\$140 his tax

Illustrative Example 2. When the tax rate is stated as a certain per cent:

Baker's property is assessed at \$5,000; the tax rate is 1.5%. Find his tax.

$$\begin{array}{r}
 \text{SOLUTION:} \qquad \$5,000 \text{ assessed valuation} \\
 \qquad \qquad \qquad .015 \text{ tax rate} \\
 \hline
 \qquad \qquad \qquad \$75 \text{ tax}
 \end{array}$$

Illustrative Example 3. When the tax rate is stated as a certain number of dollars on each hundred of dollars.

White's property is assessed at \$3,000; the tax is \$1.75 per \$100. Find the amount of his tax.

$$\begin{array}{r}
 \text{SOLUTION:} \qquad \$3,000 = 30 \text{ hundreds of dollars} \\
 \qquad \qquad \qquad 30 \times \$1.75 = \$52.50 \text{ tax}
 \end{array}$$

Illustrative Example 4. Jones' property is assessed at \$5,000. The tax rate is \$25 per \$1,000. Find the amount of his tax.

$$\begin{array}{r}
 \text{SOLUTION:} \qquad \$5,000 = 5 \text{ thousands of dollars} \\
 \qquad \qquad \qquad 5 \times \$25 = \$125
 \end{array}$$

ORAL EXERCISES

1. State each of the following tax rates in two other ways:

2.5%

32 mills

\$1.87 per \$100

2. If a certain state assesses property at $\frac{3}{4}$ of its real value, what is the assessed value of property worth \$4,000? Of a building worth \$12,000? Of a manufacturing plant worth \$8,720?

3. If property valued at \$10,000 is assessed at $\frac{3}{4}$ of its value and the tax rate is \$2.30 per \$100, what is the tax?

4. A man pays 2% tax on $\frac{1}{2}$ of the real value of his house. What per cent of the real value does he pay?

5. Jones owns property worth \$5,000 and is taxed \$2 per hundred dollars on $\frac{4}{5}$ of the real value. Williams owns a house worth \$5,000 and is taxed 19 mills on $\frac{2}{3}$ of the real value. Which pays the larger tax? How much does he pay?

WRITTEN EXERCISES

1. Complete the following form:

REAL VALUE OF PROPERTY	FRACTION OF VALUE ASSESSED	RATE OF TAX	AMOUNT OF TAX
\$135,500	Full value	\$.004 on \$1	
230,000	" "	\$1.64 per \$100	
38,400	$\frac{3}{8}$ "	\$6.845 per \$1,000	
25,000	$\frac{3}{8}$ "	2 mills on \$1	

In a certain city, a discount of 1% is allowed on all taxes paid before Feb. 10; if paid on or after Feb. 10 and before Mar. 1, $\frac{1}{2}$ % discount is allowed; if paid on or after Mar. 1 and before Apr. 1, no discount is allowed; if paid on or after Apr. 1, $\frac{1}{2}$ % is added on the first day of each month for the remainder of the year. Find the amount of tax on each of the following in that city at \$1.95 per \$100:

2. A house assessed at \$3,000, tax paid Feb. 10.
3. A store assessed at \$10,000, tax paid Mar. 29.
4. A building assessed at \$50,000, tax paid July 25.
5. An apartment house assessed at \$75,000, tax paid Nov. 15.

In New York City one-half of the tax on real estate and all the tax on personal property are due and payable on and after May 1. The other half of the real estate tax is due and payable on and after Nov. 1. A discount of 4% per annum is allowed on the second half of the real estate tax if paid before Nov. 1, provided the first half has been paid. Interest at 7% per annum from May 1 is added to all payments of the first half of the real estate tax and all personal taxes paid on and after June 1. Interest at 7% per annum from Nov. 1 is added to all payments of the second half of the real estate tax on and after Dec. 1. Find the tax due and payable, according to these regulations, on the following:

	6	7	8	9
Property.....	Business building	Railroad	Apt. house	Lot
Assessed valuation..	\$13,000,000	\$17,850,000	\$100,000	\$3,000
Borough.....	Manhattan	Queens	Brooklyn	Richmond
Rate (1920).....	\$2.53	\$2.54	\$2.54	\$2.53
Date of Payment:				
First half.....	June 15, 1920	May 20, 1920	Sept. 5, 1920	Feb. 10, 1920
Second half...	Oct. 10, 1920	Nov. 15, 1920	Feb. 10, 1921	Sept. 1, 1920
Total Tax.....				

In another city the taxes are paid in two equal payments. Taxes are due Jan. 1 of each year. The first half becomes delinquent and draws interest at $1\frac{1}{4}\%$ per month on and after Mar. 1. The second half becomes delinquent and draws interest at $1\frac{1}{4}\%$ a month on and after Aug. 1.

If the state tax is 22¢, and the city tax is \$1.36 per \$100, find the total tax paid on the following:

10. A building valued at \$37,000, both instalments paid June 25.

11. A dwelling valued at \$15,500, first instalment paid Feb. 20; second instalment paid Nov. 25.

111. Computation of Taxes by a Table.—A table similar to the following may be prepared for any city tax rate and then used for the computation of the taxes of that city.

TAX TABLE

(Type rate 15 mills on one dollar)

PROPERTY	TAX	PROPERTY	TAX	PROPERTY	TAX	PROPERTY	TAX	PROPERTY	TAX
\$10	\$.15	\$100	\$1.50	\$1,000	\$15.00	\$10,000	\$150.00	\$100,000	\$1,500.00
20	.30	200	3.00	2,000	30.00	20,000	300.00	200,000	3,000.00
30	.45	300	4.50	3,000	45.00	30,000	450.00	300,000	4,500.00
40	.60	400	6.00	4,000	60.00	40,000	600.00	400,000	6,000.00

WRITTEN EXERCISES

Using the above table find the tax on the following:

- \$25,600 $\$25,600 = \$20,000 + \$5,000 + \$600$
- \$32,500
- \$80
- \$12,600
- \$76,800

112. How the Tax Rate Is Determined.—The assessed valuation of the property is found from the assessors' lists.

The amount of money needed, divided by the assessed valuation of the property, gives the tax rate to be levied. The following type solution shows the method and a form of finding the tax rate.

STATE TAX:

State budget.....	\$2,500,000	
Assessed value of property in state.....	\$1,250,000,000	
	$\$2,500,000 \div \$1,250,000,000 =$	.20% State rate

COUNTY TAX:

County budget.....	\$40,000	
Assessed value of property in county.....	\$5,000,000	
	$\$40,000 \div \$5,000,000 =$	.80% County rate

CITY TAX:

City budget.....	\$16,000	
Assessed value of property in city.....	\$1,280,000	
	$\$16,000 \div \$1,280,000 =$	1.25% City rate
		2.25% Total rate

ORAL EXERCISE

1. State some reasons why the tax rate will vary in different cities.

WRITTEN EXERCISES

1. Find the total tax rate on the following:

State budget.....	\$ 2,500,000	
State assessed valuation.....	1,500,000,000	
		 state rate
County budget.....	28,000	
County assessed valuation.....	4,500,000	
		 county rate
City budget.....	15,000	
City assessed valuation.....	1,500,000	
		 city rate
School budget district.....	35,000	
Assessed valuation, school.....	1,500,000	
		 school rate
		 total rate

2. In a certain city the tax rate is as follows:

State tax.....	\$.475
County "	.59
City "	1.075
School "	1.49

Find the total tax.

3. If property is assessed at $\frac{3}{4}$ of its real value and Horton owns the following property:

	REAL VALUES
House.....	\$10,000
Store.....	7,500
Personal.....	25,500

Find his total tax.

4. Find the amount of his tax for the state. The county. The city. The schools.
5. What per cent of the total tax in Exercise 2 applies to each division?

113. Inheritance Taxes. An inheritance tax is a tax on the property of a deceased person. This is assessed in the greater number of the states but varies in different states, and a person interested in this subject should look up the law for each state. The following rates are for New Jersey and New York.

New Jersey. To husband or wife, child, adopted child or its issue, or lineal descendant, the rates are:

1% from \$	5,000 to \$	50,000
1½% " "	50,000 " "	150,000
2% " "	150,000 " "	250,000
3% above	250,000	
\$5,000 exempt		

To parents, brother, sister, son-in-law, and daughter-in-law, the rates are:

2%	from \$	5,000 to \$	50,000
2½%	"	50,000 "	150,000
3%	"	150,000 "	250,000
4%	above	250,000	
All others 5%			
\$5,000 exempt			

Preferred obligations:

1. Judgments
2. Funeral expenses
3. Medical expenses of last sickness

New York. If inheritance is received by father, mother, husband, wife or child, adopted child:

Exemption to amount of \$5,000
1% on amounts up to \$25,000
2% on next \$75,000
3% on next \$100,000
4% upon all additional sums

If inheritance is received by brother, sister, wife or widow of son, or husband of daughter:

Exemption to the amount of \$500
2% on amounts up to \$25,000
3% on next \$75,000
4% on the next \$100,000
5% thereafter

If inheritance is received by any person or corporation other than those above named:

Exemption to the amount of \$500
5% on amounts up to \$25,000
6% on the next \$75,000
7% on the next \$100,000
8% thereafter

Preferred obligations:

1. Funeral and administration expenses
2. Debts preferred under United States laws
3. Taxes
4. Judgments and decrees

114. Federal Inheritance Tax.—The federal tax is imposed on the estate as a whole, not on the shares of the several legatees, irrespective of the beneficiaries to the decedent.

\$50,000 of each estate is exempt from tax. The rates on the excess are as follows:

Not exceeding	\$	50,000	1%
From \$ 50,000 to		150,000	2%
“ 150,000 “		250,000	3%
“ 250,000 “		450,000	4%
“ 450,000 “		750,000	6%
“ 750,000 “		1,000,000	8%
“ 1,000,000 “		1,500,000	10%
“ 1,500,000 “		2,000,000	12%
“ 2,000,000 “		3,000,000	14%
“ 3,000,000 “		4,000,000	16%
“ 4,000,000 “		5,000,000	18%
“ 5,000,000 “		8,000,000	20%
“ 8,000,000 “		10,000,000	22%
Exceeding		10,000,000	25%

WRITTEN EXERCISES

1. Find the amount of inheritance tax to be paid to each, the state of New Jersey and the United States, on an estate of \$600,000 willed as follows: \$200,000 to decedent's wife; \$100,000 to each of three children; balance divided equally among the mother, a brother, and two sisters.

2. What would the state and federal inheritance tax amount to in New York?

3. Find the amount of inheritance tax to be deducted from each of the following amounts willed by a decedent of New Jersey:

\$40,000	to the wife
25,000	“ a son
25,000	“ a sister
10,000	“ the father
5,000	“ a brother

4. Apply Exercise 3 to New York.

115. Mortgage Tax Law of New York.—All mortgages upon real estate in New York can be made tax-exempt by paying the recording tax of $\frac{1}{2}$ of 1%. This will permanently exempt the mortgage from taxation.

WRITTEN EXERCISES

1. Find the recording tax on a mortgage of \$56,000.
2. What is the interest for the first year and the recording tax on a mortgage of \$4,800?

116. Income Tax.—The present income tax law was enacted on account of the increased expenses of the world war. The law is here given, with some of the features.

The tax shall begin at 4%, on incomes above the amount exempted up to \$4,000 normal tax, and 8% normal tax on all incomes in excess of \$4,000 with the exemption allowed. In the case of a head of a family, married man or woman, an exemption of \$2,500 for incomes up to \$5,000, and \$2,000 for those over \$5,000, and \$400 for each dependent child under 18 is allowed. The exemption for a single man or woman is \$1,000. Exemptions are allowed state and municipal paid employees. Only one deduction is allowed from the aggregate income of both husband and wife living together.

The gross income includes gains, profits, and income from any source whatever, except the interest on some United States government bonds or bonds of the political subdivisions of the United States.

The net income is obtained by deducting from the gross income all necessary expenses actually paid in carrying on the business, such as interest paid on indebtedness; loss from bad debts, if charged off; taxes; fire losses not covered by insurance or otherwise; and a reasonable depreciation on the value of the property. Personal, household, and living expenses are not included.

Illustrative Example. A man living with his wife has a net income of \$5,000. How much income tax is he required to pay?

SOLUTION:	\$5,000 net income
	2,500 exemption
	\$2,500 taxable income
	.04 normal rate
	\$ 100 income tax

117. Super-Tax or Surtax. An additional tax is assessed on large incomes in excess of a certain amount. The table below shows the additional rates charged on the portions of net income above certain stated amounts (not deducting the \$1,000 or \$2,500 or \$2,000 exemption applying to the normal tax.)

Exceeds \$						Rate	Cumulative Amounts
	6,000	but does not exceed	\$	8,000		1%	20
"	8,000	"	"	"	10,000	1	40
"	10,000	"	"	"	12,000	2	80
"	12,000	"	"	"	14,000	3	140
"	14,000	"	"	"	16,000	4	220
"	16,000	"	"	"	18,000	5	320
"	18,000	"	"	"	20,000	6	440
"	20,000	"	"	"	22,000	8	600
"	22,000	"	"	"	24,000	9	780
"	24,000	"	"	"	26,000	10	980
"	26,000	"	"	"	28,000	11	1,200
"	28,000	"	"	"	30,000	12	1,440

Exceeds \$	30,000	but	does not exceed \$	32,000	13	1,700
"	32,000	"	" " " "	34,000	15	2,000
"	34,000	"	" " " "	36,000	15	2,300
"	36,000	"	" " " "	38,000	16	2,620
"	38,000	"	" " " "	40,000	17	2,960
"	40,000	"	" " " "	42,000	18	3,320
"	42,000	"	" " " "	44,000	19	3,700
"	44,000	"	" " " "	46,000	20	4,100
"	46,000	"	" " " "	48,000	21	4,520
"	48,000	"	" " " "	50,000	22	4,960
"	50,000	"	" " " "	52,000	23	5,420
"	52,000	"	" " " "	54,000	24	5,900
"	54,000	"	" " " "	56,000	25	6,400
"	56,000	"	" " " "	58,000	26	6,920
"	58,000	"	" " " "	60,000	27	7,460
"	60,000	"	" " " "	62,000	28	8,020
"	62,000	"	" " " "	64,000	29	8,600
"	64,000	"	" " " "	66,000	30	9,200
"	66,000	"	" " " "	68,000	31	9,820
"	68,000	"	" " " "	70,000	32	10,460
"	70,000	"	" " " "	72,000	33	11,120
"	72,000	"	" " " "	74,000	34	11,800
"	74,000	"	" " " "	76,000	35	12,500
"	76,000	"	" " " "	78,000	36	13,220
"	78,000	"	" " " "	80,000	37	13,960
"	80,000	"	" " " "	82,000	38	14,720
"	82,000	"	" " " "	84,000	39	15,500
"	84,000	"	" " " "	86,000	40	16,300
"	86,000	"	" " " "	88,000	41	17,120
"	88,000	"	" " " "	90,000	42	17,960
"	90,000	"	" " " "	92,000	43	18,820
"	92,000	"	" " " "	94,000	44	19,700
"	94,000	"	" " " "	96,000	45	20,600
"	96,000	"	" " " "	98,000	46	21,520
"	98,000	"	" " " "	100,000	47	22,460
"	100,000	"	" " " "	150,000	48	46,460
"	150,000	"	" " " "	200,000	49	70,960
"	200,000	"	" " " "	300,000	50	120,960
"	300,000	"	" " " "	400,000	50	220,960
"	500,000	"	" " " "	1,000,000	50	470,960
"	1,000,000	"	" " " "		50	

Illustrative Example: Jones, a married man with no children, receives a salary of \$12,000 a year but has no other income. Find his total income tax.

SOLUTION:

NORMAL TAX

\$12,000 net income	
2,500 exemption	
\$9,500 taxable income (normal tax)	
\$4,000 at 4%	= \$160
\$5,500 " 8%	= 440

SURTAX

From \$ 6,000 to \$10,000	\$4,000 at 1%	= \$ 40
" 10,000 " 12,000	2,000 " 2%	= 40
		\$ 80

TOTAL TAX

$$\$160 + \$440 + \$80 = \$680$$

Credits for determining normal tax:

1. Dividends from corporations which are subject to the tax upon net income.
2. Interest from United States Liberty bonds.
3. Single persons have an exemption of \$1,000; married persons or heads of families have exemptions of \$2,000, but the total exemption of both husband and wife shall not exceed \$2,000.
4. An additional credit of \$200 is allowed each head of a family for each dependent under 18 years of age or incapable of self-support because mentally or physically defective.

Credits for determining surtaxes:

1. Income from a total of \$5,000 par value invested in Second, Third, and Fourth Liberty Loan bonds.

2. In addition to the above, the income from \$30,000 par value of Fourth Liberty Loan bonds will be exempt from surtaxes until 2 yr. after the termination of the war. The income from an amount of Second and Third Liberty Loan bonds not exceeding one and one-half times the amount of bonds of the Fourth Liberty Loan originally subscribed for and still owned at the time of making return, but not to exceed a total of \$45,000 par value, will be exempt from surtaxes until 2 yr. after the war. And in addition, the income from \$30,000 par value of Fourth Liberty Loan bonds converted from the First Liberty 3½'s will also be exempt from surtaxes until 2 yr. after the war.

3. Income from Federal Farm Loan bonds will be tax-free.

Illustrative Example. To find the income tax on an income of \$50,000 made up as follows:

INCOME TO BE REPORTED BUT EXEMPT FROM TAX

Income from municipal bonds.....	\$15,300
Interest on 3½% Liberty's.....	1,050
	\$16,350

INCOME WHICH MUST BE REPORTED

Salary.....	\$10,000
Interest from real estate, mortgages, and rents.....	15,000
Corporation dividends.....	1,000
Interest on Liberty 4's and 4½'s issued previous to the 4th Loan. (The owner originally subscribed for \$30,000 of the 4th Loan and still holds them).....	1,250
Interest on railroad and utility bonds not tax-free.....	6,400
	\$33,650

TO DETERMINE NORMAL TAX

Net income subject to tax.....		\$33,650
Less Credits:		
1. Corporation dividends.....	\$1,000	
2. Interest on Liberty bonds.....	1,250	
3. Fixed exemption for married person.....	2,000	
4. One dependent child.....	400	4,650
Income subject to normal tax.....		\$29,000

TO DETERMINE SURTAX

Net income.....		\$33,650
Less Credits:		
A and B income from Liberty bonds.....		1,250
Income subject to surtax.....		\$32,400
Accumulated surtax at various rates on \$32,000.....		\$1,700
Surtax on \$400 at 15%.....		60
Total surtax.....		\$1,760
Subject to 4% rate, \$4,000.....	\$ 160	
Subject to 8% rate, \$25,000.....	2,000	
Normal Tax.....	\$2,160	
Surtax.....	1,760	
Total tax.....	\$3,920	

The interest on \$1,250 (A and B) might be considered the following year.

WRITTEN EXERCISES

1. A single man has a salary of \$8,000 a year and no other income. Find his total income tax.
2. A married man with no children has a yearly income of \$8,000 in salary and no other income. Find his income tax.
3. A single man has a yearly salary of \$20,000 and no other income. Find his total income tax.

4. A married man has a yearly income of \$20,000 salary. He has one dependent child. What is his total income tax?

5. Find the net income and the total income tax for a married man with no children, from the following data:

Salary \$3,200; interest on money loaned by him in the form of a note, \$500; rent on buildings owned by him, \$1,400.

He pays \$1,200 interest on money which he has borrowed; repairs, insurance, and depreciation on his rented property \$175; state and local taxes \$225.

6. Find the net income and the total tax for a married man from the following information:

Cost of goods sold during the year \$135,000. Gross sales \$185,000.

Wages of employees, insurance, rent, and other business expenses \$12,000. Loss from bad debts, determined and charged off \$895.

He had borrowed \$4,000, on which he paid a year's interest at 6%. His store building was worth \$7,000, and he estimated the annual depreciation at 2% of the value of the building.

Net Sales = Gross Sales — Returned Sales

Gross Profit = Net Sales — Cost of Goods Sold

Cost of Goods Sold = Net Sales — Gross Profit

Net Profit = Gross Profit — Expenses

7. A married man with no children has an income of \$80,000. Find the income tax made up as follows:

INCOME TO BE REPORTED BUT EXEMPT FROM TAX

Income from Municipal bonds.....	\$20,000
Interest on 3½ Liberty's.....	2,000
	\$22,000

INCOME WHICH MUST BE REPORTED

Salary.....	\$20,000
Interest from real estate and rent.....	18,000
Interest on Liberty 4's and 4½'s issued previous to the 4th Loan. (The owner originally subscribed for \$30,000 of the 4th Loan and still holds them).....	2,000
Interest on railroad bonds not tax-free.....	18,000
	\$58,000

CHAPTER IX

INTEREST ON BANK ACCOUNTS

118. Bank Interest.—Every business banks its money and if the amount on deposit is large or is not required for current use, interest is usually earned thereon. The computing of the amount of interest earned is often a difficult problem, the difficulty being in part due to the different methods of paying interest used by different banks and in part to the fact that the amount on deposit is often changing, owing to the frequent deposit and withdrawal of cash.

119. Kinds of Banks.—Money may be deposited in a savings bank, a postal savings bank, or an ordinary commercial bank.

A **savings bank** is primarily for the purpose of accepting deposits from persons who wish to put their savings in an institution which shall guarantee them a certain rate per cent upon their money. These banks are chartered by the state and are under the supervision of the state banking department. Common rates of interest paid by these banks are 3%, 3½%, and 4%. Money in these banks cannot be drawn out, except by a check payable to the depositor himself. The law allows these banks the privilege of requiring from 15 to 60 days' notice of withdrawal, but this is seldom exercised.

A **postal savings bank** is one conducted by the United States government, in which savings may be deposited where they will draw a small rate of interest from the government.

A **commercial** bank is one which accepts deposits which are subject to check at any time and to any person or firm. These banks are under the control and supervision of the state banking department, or, in the case of national banks, the supervision of the United States Treasury Department.

120. Savings Banks.—The interest term in these banks is the period of time between the dates on which interest payments are due; i.e., if the interest payments are due Jan. 1 and July 1, the interest term is 6 mo. If due Jan. 1, Apr. 1, July 1, and Oct. 1, the interest term is 3 mo., etc. In some savings banks deposits begin to draw interest from the first of each month. In most banks, however, only such sums as have been on deposit for the full term may draw interest.

Interest is computed on the number of dollars, and the parts of a dollar are not considered. When the interest is due it may be withdrawn, or left in the bank, in which case it is added to the balance and draws interest as any deposit. Savings banks, therefore, pay compound interest. In computing savings bank interest it is important that the form of the solution and the form of the work be carried along together if one desires to make the work simple. The two forms below will show the solution of the following example, and the explanation at the end should also be read as the computer works over the example.

Illustrative Example. The interest days of the Franklin Savings Bank are Jan. 1, Apr. 1, July 1, and Oct. 1. On each of these interest days, interest at the rate of 4% per annum is computed on the smallest quarterly balance.

The account of J. B. Jackson follows:

J. B. JACKSON

DATE	DEPOSITS	INTEREST	WITHDRAWALS	BALANCE
1920				
Jan. 18.....	\$200			\$200.00
Apr. 15.....	250			450.00
May 2.....			\$100	350.00
June 1.....	50			400.00
July 1.....		\$2.00		402.00
July 16.....	150			552.00
Aug. 25.....	100			652.00
Oct. 1.....		4.02		656.02
Nov. 12.....			75	581.02
1921				
Jan. 1.....		5 81		586.83

Second form accompanying the above form:

	SMALLEST BALANCE	QUARTERLY INTEREST
First quarter.....	\$ 0.00	\$0.00
Second "	200.00	2.00
Third "	402.00	4.02
Fourth "	581.02	5.81

EXPLANATION: The first interest term was from Jan. 1 to Apr. 1; but since Mr. Jackson made no deposit until Jan. 18, the smallest balance on deposit during the entire interest term was \$0.00, and therefore no interest is added Apr. 1.

The second interest term was from Apr. 1 to July 1. The smallest balance on deposit during the entire term was \$200. The interest was \$2.

The third interest term was from July 1 to Oct. 1, the smallest balance being \$402. The interest was \$4.02.

The fourth term was from Oct. 1 to Jan. 1, the smallest balance being \$581.02. The interest was \$5.81.

121. Other Methods of Computing Interest.—Some savings banks compute the interest on monthly balances, some on quarterly balances, and some on semiannual

balances. Some banks add the interest quarterly, and some add it semiannually.

The following illustrations will make these principles clear. The explanation form should be carried along with the other form.

Illustrative Example. Suppose that the above-mentioned bank computed the interest on quarterly balances, but added the interest semiannually—we would then have:

EXPLANATION FORM:

	SMALLEST BALANCE	QUARTERLY INTEREST	SEMIANNUAL DIVIDEND OF INTEREST
First quarter.....	\$ 0.00		
Second “	200.00	\$2.00	\$2.00
Third “	402.00	4.02	
Fourth “	577.00	5.77	9.79

Then the account would read as follows:

J. B. JACKSON

DATE	DEPOSITS	INTEREST	WITHDRAWALS	BALANCE
1920				
Jan. 18.....	\$200			\$200.00
Apr. 15.....	250			450.00
May 2.....			\$100	350.00
June 1.....	50			400.00
July 1.....		\$2.00		402.00
July 16.....	150			552.00
Aug. 25.....	100			652.00
Nov. 12.....			75	577.00
1921				
Jan. 1.....		9.79		586.79

In case the bank computed the interest on the monthly balances and added interest quarterly, the explanation form of Jackson's account would appear thus:

J. B. JACKSON

MONTH	MONTHLY BALANCE	1 MONTH INTEREST $\frac{1}{4}$ OF 1%	QUARTERLY DIVIDEND OF INTEREST
1920			
Jan.....	\$ 0.00	\$0.00	
Feb.....	200.00	.66	
Mar.....	200.00	.66	\$1.32
Apr.....	201.32	\$.67	
May.....	351.32	1.17	
June.....	401.32	1.33	3.17
July.....	404.49	\$1.34	
Aug.....	554.49	1.84	
Sept.....	654.49	2.18	5.36
Oct.....	659.85	\$2.19	
Nov.....	584.85	1.94	
Dec.....	584.85	1.94	6.07
1921			
Jan.....	590.92		

NOTE: The cents in the principal are dropped, and the parts of cents in the interest are also dropped in this work.

WRITTEN EXERCISES

1. Make accounts for E. C. Stewart to and including Jan. 1, 1921, one for each of the following methods of interest at 4%:

- Interest computed quarterly and added quarterly.
- Interest computed quarterly and added semiannually.
- Interest computed monthly and added quarterly.

DATE	DEPOSITS	WITHDRAWALS
1920		
Jan. 19.....	\$400	
Feb. 13.....	500	
Mar. 5.....		\$ 50
Mar. 17.....	75	
June 6.....	150	
July 25.....		75
Aug. 3.....	200	
Oct. 15.....	125	
Nov. 21.....		100

2. Apply the methods of Exercise 1 to the following data and make an account with J. H. Harris.

DATE	DEPOSITS	WITHDRAWALS
1919		
Apr. 1.....	\$128	
May 15.....	85	
July 3.....		\$30
Oct. 3.....	60	
Dec. 22.....	75	
1920		
June 1.....		50
Aug. 12.....	100	
1921		
Mar. 15.....	175	

Find balance July 1, 1921.

3. Using 4%, computed quarterly and added semiannually, find from the following data the balance Jan. 1, 1921:

Balance Jan. 1, 1919, \$400.50; deposited Sept. 15, 1919, \$250; withdrew May 15, 1920, \$100; deposited July 1, 1920, \$75, withdrew Oct. 1, 1920, \$60.

4. Copy and complete the following account, supplying the missing amounts. Interest days, Jan. 1, Apr. 1, July 1, and Oct. 1. Rate 4%.

AMERICAN SAVINGS BANK

IN ACCOUNT WITH MR. JOSEPH B. OLIVER

DATE	DEPOSITS	INTEREST	WITHDRAWALS	BALANCE
1920				
Jan. 1.....	\$250.00			
Mar. 8.....	350.00			
Mar. 28.....			\$100	
Apr. 15.....	125.50			
July 1.....	75.00			
Aug. 13.....			125	
Sept. 15.....	60.25			
Oct. 1.....	150.00			
Dec. 5.....			50	

122. Postal Savings Banks.—No person can deposit less than \$1 nor more than \$100 a month in these banks, nor can he have a balance at any time of more than \$500, exclusive of interest. Interest is allowed at the rate of 2% per year, for **each full year** that the money remains on deposit, beginning with the first day of the month following that in which it is deposited.

A depositor may exchange his certificates, under definite conditions, in multiples of \$20, for United States government registered or coupon bonds paying $2\frac{1}{2}\%$, but this is to be done at the beginning of the year if such bonds are available at that time. These may be held in addition to the \$500 mentioned above.

Money may be withdrawn if one gives up to the postal officer, where the deposit was made, the savings certificates for the withdrawal amount.

Illustrative Example 1. How much interest would I receive in a postal savings bank, if I deposited \$8 Jan. 1, 1920, and withdrew it Jan. 1, 1921?

SOLUTION: The answer is, none, because the money does not begin to draw interest until Feb. 1, 1920.

Illustrative Example 2. How much interest will I receive if I deposit \$15 on Jan. 1, 1920, and withdraw it on July 1, 1921?

SOLUTION: Since the money is in the bank 1 full year but not all of the 2nd year, I would receive only 1 year's interest or 2% of \$15 = \$.30.

123. Computation of Depositors' Daily Balances.—The following method is one employed by some banks to compute a depositor's daily balance on a checking account. It will be noted that from Oct. 31 to Nov. 2 there were 2 da. upon which the depositor had 1 thousand dollars on deposit. This means the same in value as 2 thousand dollars for 1 da. Again on Nov. 8, there had been 7 da. on

which the deposits had not fallen below 2 hundred dollars. This is equivalent to 14 hundred or 1 thousand, 4 hundred dollars for 1 da. The account may thus be carried along for the month and at the end of the month the aggregate for 1 da. may be found. The number of hundreds is then dropped, and the interest on the aggregate at the given per cent may be found as follows:

$$\$50,000 \text{ at } 2\% = \$1,000$$

$$\$1,000 \div 365 = \$2.74, \text{ amount of interest to be credited to the account.}$$

H. N. TRUST COMPANY
STATEMENT OF INTEREST
ACCOUNT OF B. H. JONES

DATE	DAYS	BALANCE		AGGREGATE		RATE	INTEREST
		Thous.	Hund.	Thous.	Hund.		
Oct. 31.....	2		5	1			
Nov. 2.....	2	1		2			
" 4.....	4		6	2	4		
" 8.....	7		2	1	4		
" 15.....	2	2	3	4	6		
" 17.....	13	3		30			
" 30.....							
				50	14	2%	\$2.74

WRITTEN EXERCISES

1. How much interest would I receive if I should deposit \$25 in a postal savings bank Jan. 1, and withdraw it 4 mo. later?
2. How much would I receive if I should withdraw it 1 yr. and 1 mo. later?
3. I make the following deposits in a postal savings bank:

June 1, 1921.....	\$10.00
July 15, 1921.....	8.00
Sept. 1, 1921.....	15.00
Nov. 10, 1921.....	20.00

When would I be able to receive a full year's interest on each of these deposits, and how much interest would I receive for all?

4. If a man has on deposit in a postal savings bank on Jan. 1, \$125, and he decides to withdraw \$120 and purchase United States bonds bearing $2\frac{1}{2}\%$ interest, how much will all his money have earned in interest for him 1 yr. later?

CHAPTER X

BUILDING AND LOAN ASSOCIATIONS

124. Building and Loan Associations.—A building and loan association is a private corporation whose function or purpose is to assist its members in financing the building or buying of their homes. Each member agrees to purchase one or more shares of stock, and to pay for them at the usual rate of 25¢ a week or \$1 per month for each share. The money so obtained is loaned at the legal rate of interest to members wishing to buy or build homes.

The corporation receives as the profits of the association the interest on loans, fines from its members who fail to pay their dues at the specified time, premiums on loans (i.e., some associations require their members to bid for a loan—the member may make an offer of a bonus of say \$50 more than the legal rate of interest for the first year, to gain the privilege of having the money loaned to him at that time, while another member may bid only \$25, and thus fail to obtain the loan at that particular time), and the difference between the book value and the withdrawal value of members who must leave the association.

The **book value** is the actual value of the money paid in, plus all accumulated profits in which that money shall share.

The **withdrawal value** is the actual value less a certain per cent, which is determined by each association. It generally runs about 90% of the actual book value. That is, if a person withdraws he is unable to obtain as much as the actual book value of his shares at that time.

The distribution of profits must naturally be computed in order that each share shall have its pro rata share of these profits added to the amount of money paid in by the owner of each share, to obtain the actual book value. This is also necessary for the annual reports of the association, which may be required by its members, or by the state banking department, or by both.

125. The Series Plan.—This plan is to sell whatever number of shares the association shall deem necessary, say on Jan. 1; then on Apr. 1, to sell another series of shares which shall mature 3 mo. later than the first series; then another series July 1, and another Oct. 1. These may be issued but twice or even once a year, if the association thinks best or if there is little call for money for building purposes. Earnings are usually determined and divided semiannually. There are three types of problems which are of interest to the average person in an association, or connected with it as an employee or director.

126. To Find the Withdrawal Value.—

Illustrative Example. A man owns 10 shares in a building and loan association and has paid his dues at the rate of \$1 per mo. for each share for 5 yr., when he is compelled to withdraw. If he is allowed profits at 5% per annum, to what amount is he entitled?

EXPLANATION: On 10 shares at \$1 per month for each share, the dues would amount to \$600 (5 yr. = 60 mo.). The first dues (\$10) have earned profits for 60 mo., the second dues (\$10) have earned profits for 59 mo., and so on until the last dues have earned profits for 1 mo. This gives an arithmetical series of numbers which has 60 for the first term and 1 for the last term, and in which the number of terms is 60. The algebraic sum of such a series of numbers is equal to the sum of the first and last terms multiplied by one-half the number of terms, or in this example $(1 + 60) \times (60 \div 2) = 1,830$. The general method of comput-

ing the interest is to calculate it on the total dues paid in for the average time. The average time is obtained by dividing the total number of months, 1,830, by 60, the number of months in which dues have been paid, which equals $30\frac{1}{2}$. The interest on \$600 for $30\frac{1}{2}$ mo. at 5% is \$76.25, which added to the amount paid in in dues gives \$676.25, the withdrawal value.

The average time may be easily calculated by taking one-half the number of months and adding $\frac{1}{2}$ mo. to it.

SOLUTION:

$$\begin{aligned} 60 \times \$10 &= \$600, \text{ total amount paid in, in dues} \\ \$600 \times \frac{30\frac{1}{2}}{12} \times .05 &= \$76.25, \text{ profits} \\ \$600 + \$76.25 &= \$676.25, \text{ withdrawal value} \end{aligned}$$

WRITTEN EXERCISES

1. A man has paid dues of \$1 per mo. per share on 20 shares for 6 yr. when he is compelled to withdraw from an association. If the profits were 4% per annum, to what amount is he entitled?

2. State reasons why a person might be compelled to withdraw from a building and loan association.

3. A man at the end of 7 yr. finds that he must withdraw from an association. He has carried 15 shares at \$1 per mo. per share. He has unpaid fines against him of \$4.50. If his profits are calculated at $4\frac{1}{2}\%$, and all unpaid fines are deducted, find his withdrawal value.

4. A man holds 5 shares in an association that pays $5\frac{1}{2}\%$. He has been in for 8 yr. and has paid \$1 per share per month. Find the amount to which he is entitled if he withdraws.

5. If I own 12 shares in a building and loan association and must withdraw at the end of 5 yr. and 6 mo., to how much will I be entitled if the association allows 6%?

127. Computation of Profits on Shares.—When the dues and profits combined amount to the par value of the stock (usually \$200), all shares are canceled if the borrower has built or purchased a house, or each member is paid in cash if he has not borrowed from the association. To be able to know when each share shall amount to \$200 with the profits

added to the amount of money paid in by the member, it is necessary to know how to compute the profits for each share of each series. This is done in the following manner:

Illustrative Example. An association has issued 4 series of shares as follows:

1st series of 500 shares, dated Jan. 1, 1919
2d " " 400 " " July 1, 1919
3d " " 300 " " Jan. 1, 1920
4th " " 400 " " July 1, 1920

The dues in each series were \$1 per share per month. If the entire profits on Jan. 1, 1921 were \$2,765, what would be the value of 1 share of each series at that time?

EXPLANATION: Dues of \$1 a month have been paid on each of the 500 shares for 24 mo., in the first series (the average time is $12\frac{1}{2}$), which makes an average investment of $\$24 \times 500 \times 12\frac{1}{2}$, or \$150,000 for 1 mo. In the same manner the average investments in the other series are found and the total for 1 mo. is \$250,200. The share of the profits belonging to each series is in the same ratio as these average investments. The profit for 1 share in each series is obtained by dividing the number of shares in a series into the entire profit of each series. The value of 1 share in each series is the sum of all dues paid in on that share and the profit for that share.

FORM OF SOLUTION:

$\$24 \times 500 \times 12\frac{1}{2} = \$150,000$	1st series invest. for 1 mo.
$18 \times 400 \times 9\frac{1}{2} = 68,400$	2d " " " 1 "
$12 \times 300 \times 6\frac{1}{2} = 23,400$	3d " " " 1 "
$6 \times 400 \times 3\frac{1}{2} = 8,400$	4th " " " 1 "
\$250,200, total	" " " 1 "

$\frac{150,000}{250,200}$	of \$2,765 = \$1,657.673, share of 1st series
$\frac{68,400}{250,200}$	of 2,765 = 755.899, " " 2d "
$\frac{23,400}{250,200}$	of 2,765 = 258.597, " " 3d "
$\frac{8,400}{250,200}$	of 2,765 = 92.831, " " 4th "

$$\$1,657.673 \div 500 = \$3.315, \text{ profit on 1 share of 1st series}$$

$$755.899 \div 400 = 1.889, \quad " \quad " \quad 1 \quad " \quad " \quad 2d \quad "$$

$$258.597 \div 300 = .861, \quad " \quad " \quad 1 \quad " \quad " \quad 3d \quad "$$

$$92.831 \div 400 = .232, \quad " \quad " \quad 1 \quad " \quad " \quad 4th \quad "$$

$$\$24 + \$3.315 = \$27.315, \text{ value of 1 share of 1st series}$$

$$18 + 1.889 = 19.889, \quad " \quad " \quad 1 \quad " \quad " \quad 2d \quad "$$

$$12 + .861 = 12.861, \quad " \quad " \quad 1 \quad " \quad " \quad 3d \quad "$$

$$6 + .232 = 6.232, \quad " \quad " \quad 1 \quad " \quad " \quad 4th \quad "$$

One of the largest building and loan associations in the United States gives their method of computing profits as follows:

Illustrative Example. An association having 3 series of 100 shares each and profits of \$100.

SERIES	SHARES	YEARS	HALF YEARS
1	100	$\times 3 = 300$	$\times 1\frac{1}{2} = 450$
2	100	$\times 2 = 200$	$\times 1 = 200$
3	100	$\times 1 = 100$	$\times \frac{1}{2} = 50$
700			

Computation of the profits per share:

$$(\$100 \times \frac{450}{700} = \$64.284) \div 100 = \$.642 \text{ profit per share}$$

$$(\ 100 \times \frac{200}{700} = 28.572) \div 100 = \$.285 \quad " \quad " \quad "$$

$$(\ 100 \times \frac{50}{700} = 7.144) \div 100 = .071 \quad " \quad " \quad "$$

\$100

Another association uses the following plan: The profits of the association are divided and ascertained at the end of each fiscal year on the partnership plan, in the following manner: Each series investment, being the amount paid in for dues, is multiplied by the average time invested and the results added together for a sum of results. Each result is multiplied by the total earnings of the association from its institution to date, and this product divided by the sum of the results, the quotient in each case will show each series

share of the net earnings. Add the net earnings in each series to the principal of that series investment and divide the sum by the number of shares outstanding in such series, and the result will be the net result of each share in such series.

Illustrative Example. First series 3 yr. old; second series 1 yr. old.

SHARES	MONTHS	PRINCIPAL INVEST.	AVERAGE TIME	RESULTS
1st, 2,500	$\times 36 =$	\$90,000	$\times 1\frac{1}{2} =$	\$135,000
2d, 1,000	$\times 12 =$	12,000	$\times \frac{1}{2} =$	6,000
Invested,		\$102,000		\$141,090
Net Assets,		115,000		
Net Profits,		\$ 13,000		
$(\$135,000 \times 13,000 = 1,755,000,000) \div 141,000 =$ \$12,446.80 1st series profits 90,000.00				
2,500)\$102,446.80 \$40.97 1st series shares				
$(6,000 \times \$13,000 = 78,000,000) \div 141,000 =$ 553.20 2d series profits 12,000.00				
1,000)\$12,545.20 \$12.55 2d series shares				

WRITTEN EXERCISES

1. A building and loan association issued a new series at the beginning of each year. The 1st series has 300 shares, the 2d 500, the 3d 400, and the 4th 500. If the dues are \$1 per month per share and the profits at the end of the 5th year are \$4,500, find the value of 1 share in each series at the end of the 5th year.

2. Try this out with the second plan mentioned above and compare your results.

3. Try the first plan on the second plan example, and compare results. Which appears to be the better plan for the members of the association?

128. Distribution of Profits Statement.—It is necessary for an association to publish such a statement at certain intervals, either semiannually or annually. The following plan, used by some associations, will give a member a comprehensive idea of the standing of the association.

Illustrative Example. Suppose that a new series is opened each 3 mo., with dues 25¢ per wk. per share. Series number 49 has been open 520 wk. and there are 1,225 shares in the series Dec. 31, 1920. The total subscriptions paid in on series number 49 is equal to 25¢ per wk. for 520 wk., or \$130 per share, and on 1,225 shares \$159,250. During 1920 the subscriptions equal \$15,925 (52 wk. at 25¢ on each share of 1,225 shares). Subtracting \$15,925 from \$159,250 gives \$143,325, the total subscriptions paid in, Dec. 31, 1919, and this amount earns profits for all of the year 1920. The profits for 1920 are paid on one-half of the subscriptions paid in, in 1920, or on \$7,962.50, and this added to \$143,325 gives \$151,287.50, the total amount on which profits are allowed for 1920 in series 49. The per cent of profits is found by dividing the total profit on all the series by the total subscriptions in all the series sharing in profits, as shown by the total of the column headed "Total Profit-Sharing Subscriptions, Dec. 31, 1920." Allow the same rate on all series.

SERIES NUMBER	WEEKS	SHARES	SUBSCRIPTIONS IN FULL DEC. 31, 1920	SUBSCRIPTIONS FOR 1920	SUBSCRIPTIONS TO DEC. 31, 1919	INCREASE IN PROFIT-SHARING SUBSCRIPTIONS 1920	TOTAL PROFIT-SHARING SUBSCRIPTIONS DEC. 31, 1920	RATE PER CENT OF PROFITS	PROFIT PER SERIES
49	520	1,225	\$159,250	\$15,925	\$143,325	\$7,962.50	\$151,287.50	6%	\$9,077.25
50	507	1,447							
51	494	1,656							
52	481	1,740							
53	468	1,736							
54	455	1,292							
55	442	1,714							

WRITTEN EXERCISES

1. Using the same rate per cent of profits, complete the above form.
2. Given the 1st series 4 yr. old and having 3,000 shares, the 2nd series 3 yr. old and having 1,200 shares, the 3rd series 2 yr. old and having 1,500 shares, and the 4th series 1 yr. old and having 2,400 shares, net assets \$262,080, find:
 - (a) Net profits
 - (b) First series profits
 - (c) Value of 1st series shares
 - (d) Second series profits
 - (e) Value of 2nd series shares

CHAPTER XI

GRAPHICAL REPRESENTATION¹

129. Why Graphs Are Used.—The object of presenting statistical data in graphic form is to enable the reader to interpret more readily the facts contained in the collected data, and to draw proper conclusions from these facts as presented. Graphs or diagrams do not add anything to the meaning of statistics, but when drawn and studied intelligently they bring to view more clearly the various parts of a group of facts in relation to one another and to the whole group, or show effectively the fluctuations or trend characteristic of the data under consideration.

The construction of graphs is essentially mathematical and for that reason has been emphasized in this book. Moreover, some graphs are of material help in the work of bookkeeping and accounting.

The graph is designed to show:

1. The true proportion of the component parts of a group total.
2. The relation of one part of a group to other parts of the same group.

¹ Teachers are advised to study the following books if they wish to make a further study of graphical representation: U. S. Census Bureau, *United States Statistical Atlas*, 13th Census, 1910, 1914; W. C. Brinton, *Graphic Methods for Presenting Facts*, New York, The Engineering Magazine Company, 1914; A. L. Bowley, *Elementary Manual of Statistics*, New York, Charles Scribner's Sons, 1915.

3. The fluctuations or general trend in a series of similar magnitudes (or sizes), arranged date by date for a given period of time.

130. Kinds of Graphs or Charts.—Graphs used to show the true proportion of the component parts of a group total are:

1. The circle.
2. The rectangle.
3. The straight line, representing the total, and divided into segments (or sects) of proportionate lengths to represent the quantities making up the total.

Graphs used to show the relation of one part of a group to other parts of the same group are:

1. Parallel lines or bars of the same width, drawn either horizontally or vertically from a common base line.
2. Pictograms, or illustrations in perspective, showing the relative values represented by the quantities compared. This form is very unsatisfactory because the relative values are not comprehended easily by the reader.
3. Circles, squares, or any figures in which the relative values are represented in more than one dimension, the attempt being to show relative values by the size of the different circles, etc. This form is unsatisfactory because the eye cannot grasp the true relation from these sizes.

Graphs used to show the fluctuations or general trend in a series of similar magnitudes, arranged date by date for a given period of time, are:

1. The curve (Form 11), connecting points located at distances to the right of a vertical axis, as determined by the time variable, and at a distance upward from a horizontal axis, as determined by the quantity variable.
2. Comparative curves (Form 12), with a common time variable but with different quantity variable determining the location of the points in the respective graphs.

131. Construction of Graphs.—Squared (or co-ordinate) paper is necessary. Loose-leaf size is preferred for school use, since the graphs prepared should be kept as a part of the required notebook work. Paper ruled into inches and tenths of an inch will usually be found most convenient, the lines at the inches and half-inches being heavier than the rest. All rulings should be of such a color as will bring the graph into proper relief. If desired, paper with metric rulings may be used instead of that on an inch scale. A ruler with the same graduations as that of the paper can be used to advantage. Select a scale which will work well on your paper—10, 100, etc., will naturally work well on paper ruled in tenths or twentieths. The construction of curve graphs will be facilitated if the steps are completed in the following order:

1. Arrange the quantities given in the statistical table in the order of the time units, the earliest date first, and use round numbers only, e.g., use 22,000 for 22,345, etc.

2. Mark off on the horizontal axis the time points from left to right, using the vertical axis as the earliest date line.

3. Mark on the vertical axis upward from the point of intersection, the quantity scale, using the horizontal axis as

the "O" (or zero) line. The quantity scale selected is determined by the largest quantity in the series; the time scale, by the number of years or months.

4. Locate the points at the correct distance from the two axes to represent the quantities given in the series with reference to their respective dates and connect these points. The graph will then be plotted (or drawn) in the first quadrant, that is, in the space to the right of the vertical axis and above the horizontal axis.

5. Add legends to interpret the quantitative scale and time points.

6. If two or more curves are plotted on the same chart field, the procedure is the same except that a second scale, if there is one, may be indicated on the right. The curves should be distinguished by the character of the line or by color and an explanatory legend should be given.

132. Facts Shown by the Component Parts of a Circle.—

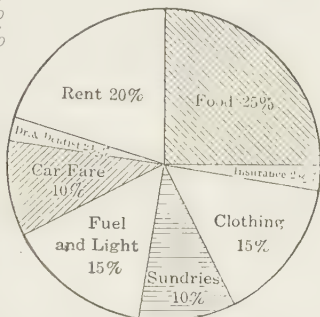
This is a form of graph quite commonly used by business men. The circle is divided into parts which will show how much of the group total is represented by each of its parts. In the following example the division of the circle is determined as follows: 15% of it is shown, representing the amount that might be used for clothing. There are 360 degrees in the whole circle. $15\% \text{ of } 360 \text{ degrees} = .15 \times 360^\circ = 54^\circ$. By the use of the protractor, which is explained in Chapter XIX, it is an easy matter to lay off 54° . The divisions should always be **from** the center of the circle.

Illustrative Example. Some good authorities claim that the following per cents are the largest per cents which should be expended out of the family income for the various expenses of the home.

Food	25%
Rent	20%
Fuel and light	15%
Clothing	15%
Carfare	10%
Sundries	10%
Doctor and dentist	2½%
Insurance	2½%

Distribution of a \$2,500 salary according to this schedule (Form 8).

Food	\$ 625.00
Rent	500.00
Fuel and light	375.00
Clothing	375.00
Carfare	250.00
Sundries	250.00
Doctor and dentist	62.50
Insurance	62.50
Total	\$2,500.00



Form 8. Circle Chart showing Distribution of Income

WRITTEN EXERCISES

1. The Adirondack park is classified by ownership by the New York State Conservation Commission as follows:

State	48%
Improved	6%
Private parks	15%
Lumber and pulp companies	23%
Private	6%
Mineral companies	2%

Show this by the above method.

2. The causes of divorce as taken from the report of a judge of the court of domestic relations in a large city were:

Disease	12%
Alcoholic drink	46%
Immorality	15%
Ill-temper and abuse	10%
Interference of parents	7%
Miscellaneous	10%

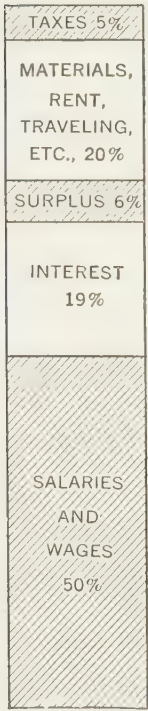
Make a chart to show these facts.

3. The payments out of the milk dollar as reported by the Borden's Farm Products Co. Inc., in 1916 were as follows:

To dairymen.....	45.87¢
To labor.....	25.41¢
To railroads.....	9.03¢
To shareholders.....	3.25¢
For materials, supplies, and expenses of bottles, boxes, etc.....	16.44¢

Chart these facts.

4. If the total sales are \$8,761, cost of the goods sold \$6,645, returns \$150, gross profit \$2,116, selling expenses \$800, and general expenses \$400, find the net profit and graph these facts in one circle.



Form 9.
Rectangle
Chart

133. Graphs by Component Parts of a Rectangle.—This plan is to divide a given rectangle up into its proportionate parts. The length of the rectangle should first be determined from the business facts, and then subdivided in proportion to the amounts which shall represent the correct parts of the total group. Paper ruled to tenths is very useful for this purpose. It is well illustrated in the following:

Illustrative Example. The gross revenue of the Bell Telephone System for 1 yr. was disposed of in the following manner:

Salaries and wages.....	50%
Interest.....	19%
Surplus.....	6%
Taxes.....	5%
Materials, rent, traveling, etc.....	20%

This is shown by the component parts of a rectangle (Form 9).

WRITTEN EXERCISES

1. Graph Exercise 1 under § 132 by this method.
2. The disposition of a 5¢ carfare paid to a certain city railroad in a year was as follows:

General expenses, including pensions and insurance.....	.223¢
Cost of power.....	.422¢
Wages and conducting transportation.....	1.34¢
Other transportation expenses.....	.128¢
Maintenance of way.....	.488¢
Maintenance of equipment.....	.36¢
Depreciation.....	.069¢
Damages and legal expenses.....	.291¢
Taxes.....	.367¢
Rentals, subways, and tunnels.....	.171¢
Interest.....	.365¢
Rental, surface lines.....	.486¢
Dividends.....	.277¢
Surplus.....	.031¢

Make a graph by the above method which will show this.

3. The utilization and accompanying waste of 1 year's coal supply for locomotives on the railroads of the United States was as follows:

	MILLIONS OF TONS OF COAL	MILLIONS OF DOLLARS
Consumed in starting fires, keeping engine hot while standing, and left in fire box at the end of run.....	18.	34.
Utilized by the boiler.....	42.	79.3
Lost in vaporizing moisture in coal.....	2.5	4.7
Lost through the company.....	.75	1.4
Lost in gases discharged from the stack.....	9.25	17.4
Lost in the form of unconsumed fuel in cinders, sparks.....	9.5	17.9
Lost through unconsumed fuel in the ash.....	3.5	6.6
Lost through radiation, leakage of steam, etc.....	4.5	8.7

HINT: Make 90 equal spaces on one side of the rectangle to represent millions of tons of coal, and 170 equal spaces on the other side to represent millions of dollars and plot by the above method.

4. Graph Exercise 2 under § 132 by the above method.

5. Make a chart of the following:

DISTRIBUTION OF RAILROAD \$100 INCOME
FOR A CERTAIN YEAR

Labor.....	\$43.20
Fuel and shop supplies.....	8.12
Material.....	16.90
Damage.....	2.22
Tax.....	4.72
Division superintendent.....	5.00
Betterment.....	1.08
Rentals.....	3.97
Interest (balance).....	

6. Graph the following data:

SOURCE OF RAILROAD \$1 INCOME FOR ONE YEAR

Passengers.....	22.2¢
Product of mines.....	23.9¢
Manufactures.....	15.1¢
Product of agriculture.....	11.7¢
Product of forests.....	7¢
Product of animals.....	4.2¢
Merchandise.....	4.2¢
Mail.....	1.9¢
Express.....	2.3¢
Miscellaneous (balance)	

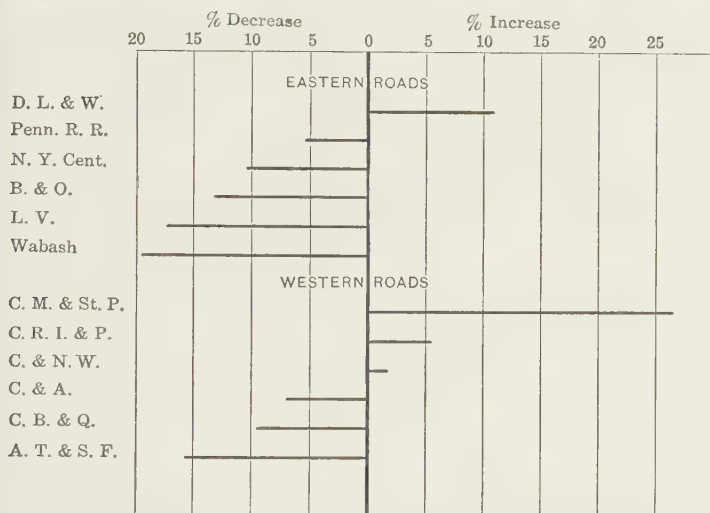
7. Graph Exercise 4 under § 132 by the above method.
8. The causes of crime as reported by a noted detective are as follows

Poverty.....	40%
Gambling and debt.....	25%
Strict parents.....	13%
Having too much money.....	1%
Opium.....	1%
Easy going parents.....	12%
Drink.....	8%

Plot this by the above method or by that of § 132.

134. Simple Comparisons by Graphs.—The graph shown in Form 10 is often better than any other form for some kinds of business facts.

Illustrative Example. The repairs and renewals of locomotives per ton tractive force for an average over 5 yr. as compared with the 4 yr. preceding this period is shown by the following graph (Form 10).



(From "Railroad Operating Costs," New York. Suffern and Son, 1911)

Form 10. A Variation of the Straight-Line Graph

WRITTEN EXERCISES

1. The distance that different kinds of trucks can travel on \$1 expenditure has been reported as follows:

5 ton Horse.....	1.6 miles	2 ton Gas.....	2.6 miles
5 " Gas.....	1.8 "	3½ " Electric....	2.75 "
3½ " Horse.....	2.2 "	2 " Horse.....	2.93 "
5 " Electric.....	2.3 "	2 " Electric....	3.3 "
3½ " Gas.....	2.4 "		

Graph this from a zero line toward the right.

2. Chart the following causes of death in a certain city for a certain year.

DISEASE	NUMBER OF DEATHS
Organic heart.....	1,325
Tuberculosis.....	1,120
Pneumonia.....	940

Bright's disease.....	640
Stomach and bowel (under 2 yr.).....	630
Apoplexy.....	560
Broncho-pneumonia.....	440
Cancer of stomach and liver.....	225
Diphtheria and croup.....	175
Alcoholism.....	140
Appendicitis.....	110
Scarlet fever.....	90

3. Make a chart of the following fluctuations in the price of eggs in the United States.

	PER CENT OF AVERAGE PER MONTH			
	1909	1910	1911	1912
Jan.....	20% Inc.	42% Inc.	41% Inc.	38% Inc.
Apr.....	20% Dec.	11% Dec.	31% Dec.	18% Dec.
July.....	12% "	15% "	32% "	22% "
Oct.....	8% "	5% Inc.	8% "	3% Inc.

4. The following table shows the amount of various foods that must be eaten to secure the same number of calories that are found in 100 grams of ordinary white bread. Make a graph to show this simple comparison.

FOOD	NO. OF GRAMS EQUIVALENT IN CALORIES TO 100 GRAMS OF WHITE BREAD	FOOD	NO. OF GRAMS EQUIVALENT IN CALORIES TO 100 GRAMS OF WHITE BREAD
White Bread.....	100	Butter.....	35
Wheat flour.....	70	Gruyere cheese.....	73
Tapioca.....	70	Smoked ham.....	74
Meat pie.....	70	Pork cutlets.....	90
Macaroni.....	70	Sliced mutton.....	90
Maize flour.....	70	Jellied fruits.....	100
Potato starch.....	75	Sirloin of beef.....	138
"War" bread.....	70	Chicken eggs.....	170
Hulled rice.....	70	Chicken.....	175
Vermicelli.....	73	Veal loin.....	180
Dried beans.....	73	Unsalted herring.....	330
Split peas.....	70	Potatoes.....	370
Noodles.....	74	Milk.....	380
Barley flour.....	70	Apples.....	500
Lentils.....	74	Mussels.....	600
Rye Bread.....	100	Spinach.....	954

5. Make a chart of the following, showing the causes of leaving positions.

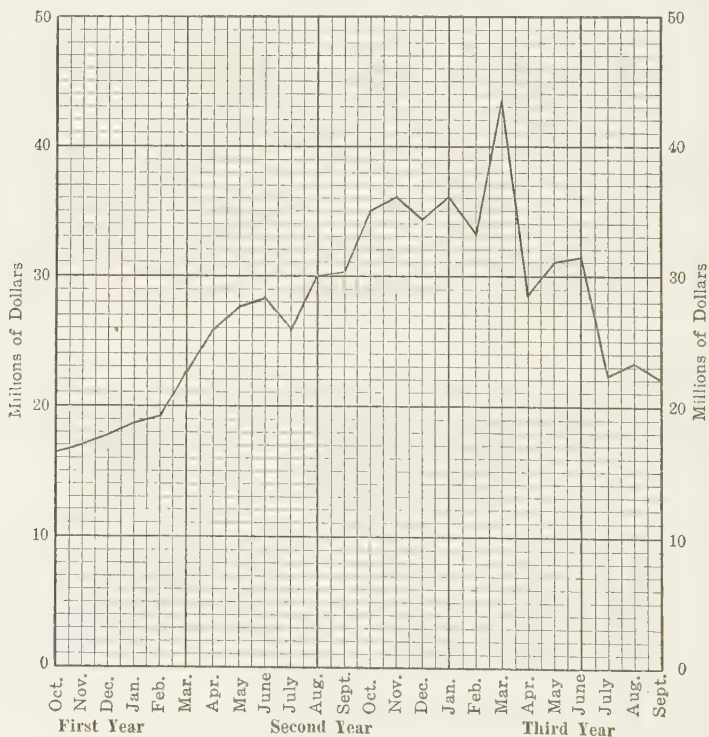
REASONS	PER CENT
Not enough money	9
Never started	6
Working conditions	20
Discharged	5
Laid off	1
Dissatisfied	2
Better job	12
Needed at home	8
Living conditions	2
Failed to report	28
Personal reasons	7

135. Curve Plotting.—This plan is considered one of the best to present business facts in such a manner that they may be easily grasped by the reader. Before we take this subject up in detail, however, it will be well to master certain well-defined rules which are very important if one is to be successful in graph work. These rules follow.

Rules for graphical representation of facts:

1. Make the title of the chart very complete and clear.
2. Make the general arrangement of a chart read from left to right.
3. The horizontal scale figures are placed at the bottom of the chart, and figures may also be used at the top if needed.
4. The figures for the vertical scale are to be placed at the left of the chart. Right-hand figures may be added if needed.
5. Include with the chart the data from which it was made.
6. Place the lettering and figures so that they may be read from the bottom or from the right-hand side of the chart.
7. Earliest date should be shown at the left and later dates to the right.
8. Charts should usually read from left to right and from the bottom to the top.
9. Green may be used to express desirable features, and red to indicate undesirable features.
10. Zero line should show on the chart whenever possible.

11. Make the zero line much heavier than the squared paper lines.
12. The bottom line should be wavy if the zero line cannot be shown.
13. If the chart refers to percentages, the 100% line should be broad like the zero line.



Form 11. Curve Graph

14. When the horizontal scale begins with zero, the vertical line at the left which represents zero should be broad.

15. If the horizontal scale denotes time, the left- and right-hand lines are wavy, as the beginning and end of time cannot be shown.

16. If curves are to be printed, be careful not to show any more lines of the co-ordinate paper than is necessary. Lines one-fourth of an inch apart are better.

17. Make the curve lines much broader than the co-ordinate ruling of the paper.

18. It is often advisable to show at the top of the chart the value of each point plotted.

19. If figures are shown at the top for each point plotted, have these figures added when possible so as to show monthly or yearly totals.

20. If a number of curves are to be shown on the same chart field, use different-colored inks, or different kinds of lines.

The graph illustration in Form 11 shows the monthly net earnings of a steel corporation over a period of 2 yr. Observe that in this graph 1 space up on the vertical axis represents 1 million of dollars, and 2 spaces on the horizontal axis represent 1 mo. of time. Note that the peak of the net earnings was in March of the 3d yr., and that the ebb (or lowest amount) of the net earnings was in October of the 1st yr. Thus any business can be pictured over a term of years. This can also be done for sales, profits, or anything that is desired.

WRITTEN EXERCISES

1. Show the rise and fall of the United States Steel Corporation's unfilled orders from the following data:

1920				1921			
Jan.....	7.75	millions of tons		Jan.....	11.5	millions of tons	
Feb.....	8	" " "		Feb.....	11.4	" " "	
Mar.....	8.5	" " "		Mar.....	11.6	" " "	
Apr.....	9.3	" " "		Apr.....	11.75	" " "	
May.....	9.8	" " "		May.....	12	" " "	
June.....	9.9	" " "		June.....	11.75	" " "	
July.....	9.6	" " "		July.....	11.3	" " "	
Aug.....	9.55	" " "		Aug.....	10.75	" " "	
Sept.....	9.6	" " "		Sept.....	10.4	" " "	
Oct.....	9.5	" " "		Oct.....	9.8	" " "	
Nov.....	10	" " "		Nov.....	9	" " "	
Dec.....	11	" " "		Dec.....	8.8	" " "	

2. Make a curve to show the rise of the New York City budget for a period of 20 yr. from the following data:

1st year.....	77	millions	11th year.....	143	millions
2d "	93	"	12th "	156	"
3d "	98.6	"	13th "	163	"
4th "	98.1	"	14th "	173	"
5th "	98.7	"	15th "	180	"
6th "	99	"	16th "	192	"
7th "	99	"	17th "	196	"
8th "	98.6	"	18th "	198	"
9th "	115	"	19th "	199	"
10th "	130	"	20th "	200	"

3. Make a curve to show the increase in the deposits of a certain national bank.

		DEPOSITS
1st year.....		\$ 4,100,000
2d "		10,600,000
3d "		14,200,000
4th "		17,500,000
5th "		21,100,000
6th "		30,200,000

4. The following annual report of a railroad is based on a certain year and is for the 4 yr. following it. Make a curve for each of the items mentioned on one chart field whose vertical column represents per cent and whose horizontal line represents years.

IN TERMS OF PER CENT

BASE YR. 1ST YR. 2ND YR. 3RD YR. 4TH YR.

1. Operating income:

Net operating revenue after

deducting taxes..... 0 39 125 113.5 147

2. Gross operating revenue..... 0 4.5 28.7 26 39

3. Operating expenses..... 0 2.3 11.1 10.4 17

5. Make a curve to show the proper inflation pressure per square inch on different-sized automobile tires. About 20 lb. is allowed to the square inch of section.

SIZE IN TIRE CROSS SECTION	INFLATION PRESSURE LB. PER SQ. IN.
3 sq. in.	60 lbs.
3½ " "	70 " "
4 " "	80 " "
4½ " "	90 " "
5 " "	95 " "
5½ " "	100 " "
6 " "	105 " "

6. Using the vertical line to represent relative value of coals in dollars, and the horizontal line to represent price of anthracite coal in dollars, and using one space (of any chosen length) for \$1 on the base line, and one-half of that space for \$1 on the vertical line, draw four lines to show the comparative values.

	PRICE	RELATIVE VALUE
Anthracite coal.....	5	5
" "	0	0
Illinois coal.....	0	0
" "	8	5
Coke.....	0	0
"	6	6.25
Pocohontas Coal.....	0	0
" "	8	9

7. Using one space (of chosen length) on the horizontal scale to represent years and one of chosen space on the vertical scale to represent \$2, show the increasing demand upon the New York City transit lines.

YEAR	PER CAPITA	YEAR	PER CAPITA
1860.....	\$ 0.00	1906.....	\$14.40
1870.....	6.00	1907.....	15.20
1880.....	8.50	1908.....	15.20
1890.....	10.00	1909.....	15.20
1900.....	12.00	1910.....	16.00
1901.....	12.40	1911.....	16.40
1902.....	12.80	1912.....	16.80
1903.....	13.20	1913.....	17.60
1904.....	13.60	1914.....	17.20
1905.....	14.00	1915.....	17.00
		1916.....	18.00

136. Comparisons Involving Time.—It is quite necessary at times to show a comparison from one year to another on the same line or article, such as sales, wages, cost of food product, etc. This is accomplished very simply by the use of the graph shown in Form 12. Note that the same amount is placed on the top side of the rectangle as on the bottom side on corresponding lines.

WRITTEN EXERCISES

1. Show the changes after 10 yr. in costs of materials and in freight rates with a graph similar to that shown in Form 12, using \$100 as the basis for each at the beginning of that time.

Labor.....	From \$100 to \$120
Interest.....	" 100 " 125
Fuel.....	" 100 " 130
Railroad rates.....	" 100 " 95
Tracks per mile.....	" 100 " 145
Rails.....	" 100 " 150
Pine lumber.....	" 100 " 183

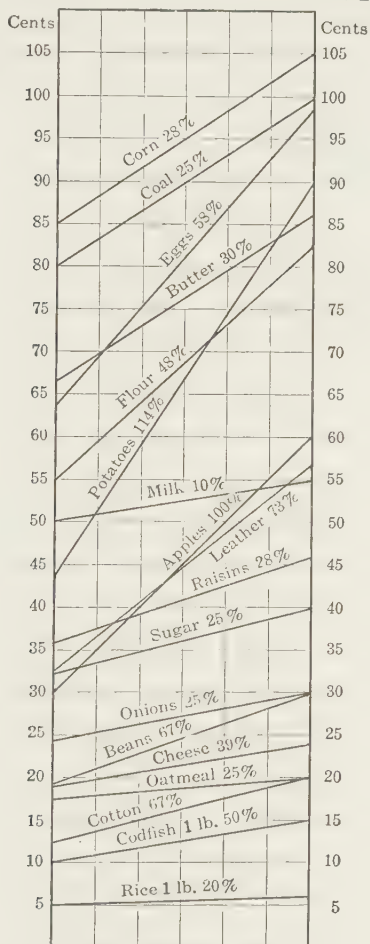
2. Show the rising wage scale of railroad labor from 1899 to 1911 to 1920 using each unit to represent \$.25 and starting with \$1 per da. in both the left- and right-hand columns.

	WAGES IN 1899	WAGES IN 1911	WAGES IN 1920
Trackmen.....	\$1.00	\$1.50	\$3.88
Station agent.....	1.75	2.20	6.96
Trainmen.....	1.95	2.95	6.40
Machinists.....	2.30	3.20	6.80
General office clerks.....	2.15	2.40	4.50
Conductors.....	3.15	4.20	7.00
Engine men.....	3.65	4.70	7.52

3. Obtain similar information concerning some railroad or large manufacturing plant and make a graph to show the results of the information which you obtain.

137. Period Charts.—It is often advisable to arrange the working hours of a number of employees in chart form.

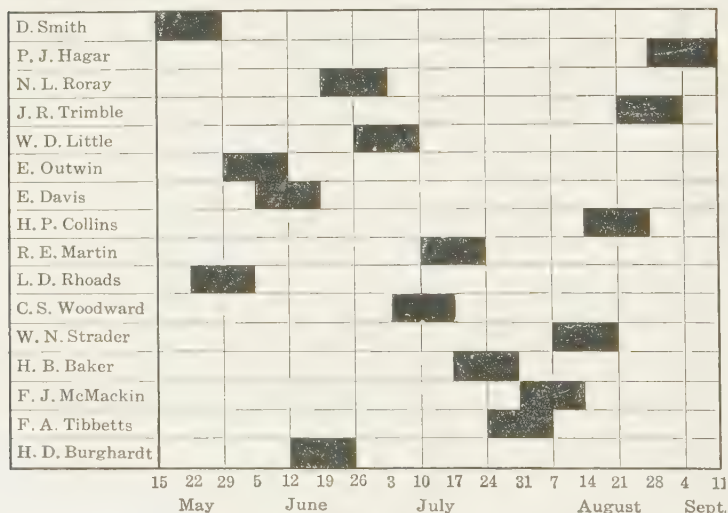
First Year Second Year



Form 12. Comparative Curves

This is particularly true when they work in shifts and thus relieve one another at stated intervals of time or at certain hours. It is also applicable in the arrangement of their vacations as will be shown in the following.

Illustrative Example. Assign 2 weeks' vacation to each of 16 men so that no more than 2 shall be away at once, and show graphically.



Form 13. Period Chart

WRITTEN EXERCISES

1. Make a chart similar to the above for 25 men.
2. Make a chart for 60 men in which no more than 3 shall be away at the same time, beginning May 17, 1920.
3. Three patrolmen in a certain city take the work from 8 to 4, 4 to 12, and 12 to 8 respectively. Each man gets 1 da. off in 27. Arrange a chart to show this. Plan one off, etc., after a certain number of days.

HINT: 1st P.
2d. P.
3d. P.

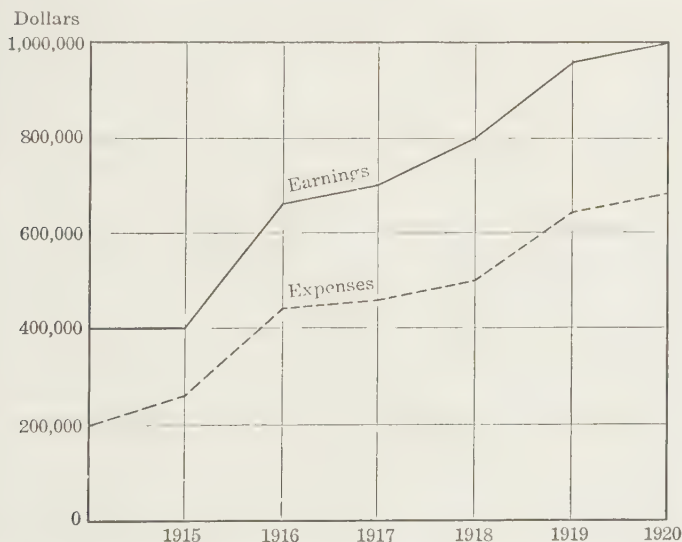
8-4	4-12	12-8

138. Comparison of Curves.—The officials of a company frequently wish to be able to see a comparison of certain items of the business. This may be easily accomplished by drawing two or more curves on the same chart field. These curves will immediately, if drawn correctly, show the comparisons desired. Form 14 is a good type of this form of graph.

Illustrative Example. The earnings and expenses of a certain railroad by years are given below:

	1915	1916	1917	1918	1919	1920
Earnings.....	\$400,000	\$660,000	\$680,000	\$800,000	\$960,000	\$1,000,000
Expenses.....	260,000	440,000	460,000	500,000	640,000	680,000

Using a solid line for the earnings and a dotted line for the expenses, these are shown on the same chart (Form 14).



Form 14. Composite Chart showing Relation between Income and Outgo
These curves are similar to the comparative curves shown in Form 12

WRITTEN EXERCISES

1. The monthly earnings and expenses of a railroad are as follows:

	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
Earnings	\$ 83,000	\$ 73,000	\$87,000	\$86,000	\$95,000	\$101,000
Expenses	57,000	61,000	63,000	61,000	70,000	73,000

	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Expenses	\$115,000	\$119,000	\$98,000	\$92,000	\$85,000	\$ 80,000
Earnings	82,000	87,000	81,000	87,000	70,000	65,000

Make two curves on the same chart to show the contrast.

2. Draw two curves to represent the average rate paid and the average number of phones in use in a city, from the following data:

	1907	1908	1909	1910	1911	1912	1913
Av. rate paid	\$0	\$120	\$113	\$100	\$85	\$69	\$62
Av. No. used	0	11,000	20,000	26,000	36,000	48,000	64,000

	1914	1915	1916	1917	1918	1919
Av. rate paid	\$60	\$56	\$52	\$46	\$40	\$39
Av. No. used	80,000	94,000	116,000	140,000	170,000	196,000

3. Representing the per cents on the vertical column and the grades on the horizontal column, make three curves to show the percentage of pupils attaining the various grades as given by three teachers in the same subject.

	40-50	50-60	60-70	70-80	80-90	90-100
1st teacher.....	4%	7%	10%	25%	30%	24%
2nd " 	5%	8%	12%	35%	25%	15%
3rd " 	3%	10%	7%	20%	30%	30%

4. Represent with four curves the outward messages from 12 selected stations of the American Telephone Company for the 12 mo. of the year.

YEAR	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1917...	13,450	13,700	13,600	13,550	13,750	13,400
1918...	14,400	14,800	15,150	14,750	14,850	14,600
1919...	15,550	15,950	15,450	15,800	15,800	15,650
1920...	16,350	16,400	15,600	16,100	16,100	16,150

YEAR	JULY	AUGUST	SEPT.	OCTOBER	NOVEMBER	DECEMBER
1917...	12,400	12,350	13,250	14,000	14,300	14,100
1918...	13,650	13,600	14,750	15,650	15,400	15,550
1919...	14,900	14,500	15,750	16,000	16,250	16,000
1920...	15,500	15,750	15,200	15,550	16,550	16,500

5. If the American Telephone Company wished to show how its business was running, it might plot the data given in Exercise 4 and draw one continuous curve for the 4-yr. period. The work can be simplified somewhat by plotting only 1 yr., and connecting these points with straight lines. Do this and note whether the line tends upward or downward. If it gradually rises what will it show about the business?

6. Construct 2 curves on the same chart to represent the debt and the stock of gold coin (in dollars) in the United States from the following data. Use solid and broken lines or two colors.

YEAR	NET DEBT OF THE UNITED STATES	STOCK OF GOLD
1860.....	50 millions	
1865.....	2,680 "	
1870.....	2,300 "	

YEAR	NET DEBT OF THE UNITED STATES	STOCK OF GOLD
1875.....	2,050 millions	110 millions
1880.....	1,900 "	340 "
1885.....	1,350 "	600 "
1890.....	900 "	700 "
1895.....	900 "	625 "
1900.....	1,100 "	1,000 "
1905.....	1,000 "	1,350 "
1910.....	1,050 "	1,600 "
1915.....	1,100 "	2,000 "
1917.....	1,150 "	3,100 "

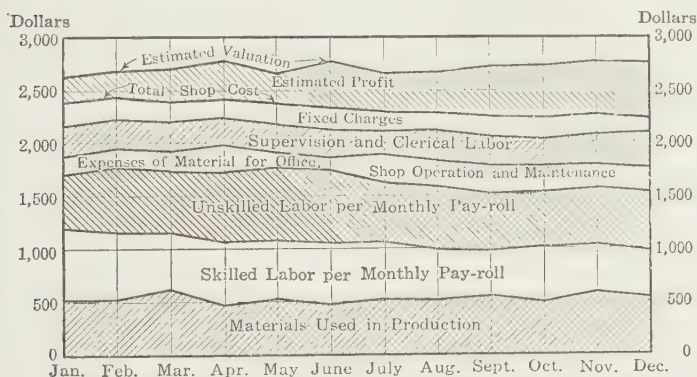
7. The following facts show advances in rentals from 1900 to 1913. They are based on a study of charges for the same properties in 48 cities having a population of 10,000 or over. Make three curves to show these. Use a solid line for the stores in the first-class business districts, a dashed line for second-class districts, and a dotted line for third-class districts, or use colors.

	1900	1901	1902	1903	1904	1905	1906
Rents 1st class.....	100	110	120	130	135	140	165
" 2nd "	100	120	130	145	155	175	190
" 3rd "	100	107	115	120	125	130	135

	1907	1908	1909	1910	1911	1912	1913
Rents 1st class.....	190	220	240	265	295	330	345
" 2nd "	215	240	255	275	297	325	340
" 3rd "	150	155	170	180	190	200	210

139. **Component Parts Shown by Curves.**—The chart illustrated in Form 15 shows the total cost plus the profit in a manufacturing business. This is of especial value to the executive, for he may see at a glance how the manufacturing end of the business is working. It will also show if there are

irregular, extra heavy expenses in some particular part of the manufacturing costs and allow him to take means to



Form 15. Chart showing Component Parts

The principles underlying the construction of this chart are similar to those discussed in connection with the rectangle chart (Form 9)

remedy them. Note that each item of the expense is added on, so that the top curve will show the total.

WRITTEN EXERCISES

1. Using the following data, construct curves similar to those shown in Form 15.

	JAN.	FEB.	MAR.	APRIL	MAY	JUNE
Materials in production	\$1,000	\$1,100	\$1,200	\$ 800	\$1,500	\$1,250
Skilled labor	1,200	1,200	1,300	1,100	1,300	1,150
Unskilled labor	1,100	1,200	1,200	1,250	1,200	1,250
Materials for office	300	300	300	350	300	400
Supervision and clerks	500	500	475	500	400	450
Fixed charges	350	350	400	400	375	380
Estimated profit	400	400	450	500	450	500

2. Make a graph showing the percentage of distribution of the expenses of operating the railroads of the United States per year. Total of all per cents should be 100.

	1890	1891	1892	1893	1894	1895	1896	1897
General Expenses.....	8	7	8	7	8	2	3	3
Maintenance of equipment.....	11	10	11	11	10	10	11	10
" " way.....	14	15	15	14	12	13	14	15
Fixed charges.....	31	30	31	30	32	33	30	31
Conducting transportation.....	36	38	35	38	38	42	42	41

	1898	1899	1900	1901	1902	1903	1904
General Expenses.....	3	2	1	1	2	2	1
Maintenance of equipment.....	12	13	13	13	13	15	14
" " way.....	15	17	16	18	18	16	16
Fixed charges.....	30	30	29	29	29	28	27
Conducting transportation.....	40	38	41	39	38	39	42

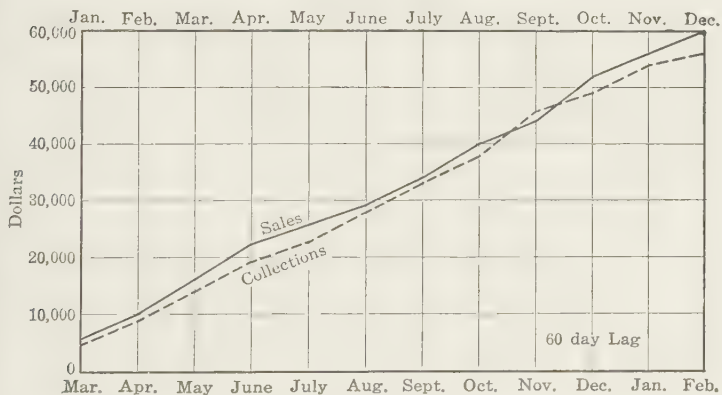
140. Correlative and Cumulative Curves. Such curves are constructed so that they show a relation as well as the total to date. For instance, in Form 16 the sales up to April are \$22,000, while the total collections to April are \$19,000. Similarly, the total sales to September are \$44,000, and the total collections to September are \$46,000. The collections for March show against the sales for January, etc. If the business conditions of the firm are ideal, the two curves should run nearly parallel to each other. A 60-da. lag means that the sales are 60 da. ahead of the collections.

WRITTEN EXERCISES

1. Construct a graph similar to that shown in Form 16, to represent the following data on production in a manufacturing plant:

MONTH	NUMBER PLANNED PER WEEK	ACTUAL OUTPUT			
		1st Week	2d Week	3d Week	4th Week
Jan.....	40	30	40	45	50
Feb.....	40	35	45	40	40
Mar.....	50	40	55	60	50
Apr.....	60	50	65	60	65
May.....	60	60	55	65	60
June.....	75	70	70	65	80
July.....	75	70	80	80	85

FOR SALES



FOR COLLECTIONS

Form 16. Correlative and Cumulative Curves

2. The following information represents the cost per car-mile of different weight trucks, as estimated by a manufacturer, as well as the actual cost. Use a dotted line for the former and a solid line for the latter (or different colors), to show the two curves.

WEIGHT OF TRUCK	ESTIMATED COST PER CAR-MILE	ACTUAL COST PER CAR-MILE
1,000 lbs.	.75¢	2.63¢
2,000 "	2.13¢	1¢
3,000 "	3.38¢	4.12¢
4,000 "	3¢	3.5¢

WEIGHT OF TRUCK	ESTIMATED COST PER CAR-MILE	ACTUAL COST PER CAR-MILE
5,000 "	3.5¢	3.8¢
6,000 "	3.75¢	4¢
7,000 "	4.25¢	5¢
8,000 "	4.5¢	6.5¢

141. Map Representations.—Map graphs² are very useful in the executive's office. They show at a glance the location of all the subsidiary offices or manufacturing plants. Form 17 shows the location of the various cantonments for the United States Army in 1918. Pins with colored heads and letters or figures can well be used with such maps. The letter could stand for the name of the city or town. A tabulation showing the names of the places opposite its corresponding number could accompany a chart.

WRITTEN EXERCISES

1. Make a map of the United States and on it locate the following selling agencies of a prominent article:

Boston, Mass.	St. Louis, Mo.	Salt Lake, Utah
Albany, N. Y.	New Orleans, La.	Butte, Mont.
New York City	Dallas, Texas	Portland, Ore.
Harrisburg, Pa.	Des Moines, Iowa	San Francisco, Cal.
Detroit, Mich.	Tulsa, Okla.	Sacramento, Cal.
Chicago, Ill.	Denver, Colo.	Milwaukee, Wis.

² Two types of maps should be used in the teaching of map graphs: the blackboard outline map for the class recitation, and the desk outline map for the notebook work by the individual pupil. Maps of the world, the United States, and your own state will be sufficient for this work. The desk outline maps should be punched to fit the notebook cover, in order that the maps prepared by the pupil may be made a part of his notebook work.

The general use of the pin map in business offices justifies the author's belief that they belong in this work. Any good wall maps mounted on cork composition and framed, together with a box of map pins of assorted colors and sizes will serve. Exercises in which pupils are required to show on the map the location of important commercial cities, industrial sections, etc., should give added interest to this subject.

2. Construct a map of New York and New Jersey and on it show the elevation of the following points on the New York, Ontario, and Western Railroad:

Weehawken.....	20 ft.	Walton.....	1,100 ft.
Cornwall.....	25 "	Northfield.....	1,700 "
Middletown.....	550 "	Oswego.....	250 "
Youngs Gap.....	1,800 "	Sidney.....	1,050 "
Oneida.....	400 "	Summit.....	1,600 "
Cadosia.....	1,000 "	Norwich.....	950 "
Apex.....	1,500 "	Eaton.....	1,350 "



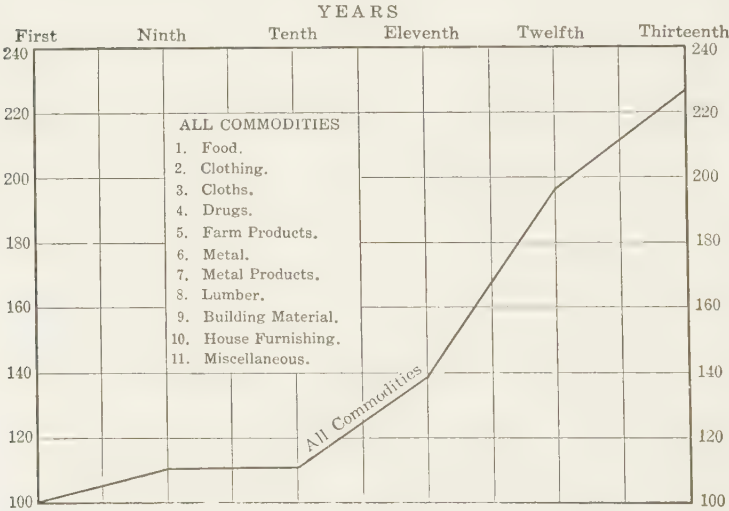
Form 17. Map Chart

142. Frequency Charts or Curves.—These curves are intended to show how often or when certain things occur. The curve given in Form 18 shows when the commodities rise and fall in price. The peak of a curve will show when the particular thing will reach the greatest amount or the greatest number of years, etc.

WRITTEN EXERCISES

1. Plot a curve to show the age at marriage of 439 ladies who were college graduates.

AGE	%	AGE	%
20.....	.5*	30.....	5.2
21.....	.7	31.....	2.8
22.....	2.8	32.....	2
23.....	5.5	33.....	2
24.....	12.5	34.....	1.1
25.....	15.2	35.....	1.1
26.....	14.1	36.....	.2
27.....	14.4	37.....	.7
28.....	11	38.....	0
29.....	8.2		



Form 18. Frequency Curve showing Changes in Costs

2. Construct a curve to show the death rate changes of Americans at various ages since 1880, as reported by an insurance company.

AGE	% DECREASE	% INCREASE
Under 20.....	17.9	
20 to 30.....	11.8	
30 to 40.....	2.3	
40 to 50.....		13.2
50 to 60.....		29.2
60 and over.....		26.4

3. Using one space on the horizontal scale to denote a 2-wk. period, and one space on the vertical scale to denote 10 deaths per thousand, construct two curves on the same chart to show the comparison of deaths of children in similar parts of two successive years in New York City, as reported by the Sheffield Milk Company, who claim their milk was sold to poor people in 15 Board of Health stations in the latter year at below cost while these people were not accorded that privilege in the former year.

	FIRST YEAR DEATHS PER 1,000	SECOND YEAR DEATHS PER 1,000
1st 2 wk.....	140	98
2d " "	123	96
3d " "	118	110
4th " "	127	120
5th " "	110	130
6th " "	130	120
7th " "	115	132
8th " "	138	131
9th " "	130	124
10th " "	110	135
11th " "	112	108
12th " "	120	118
13th " "	148	155
14th " "	200	145
15th " "	240	86
16th " "	200	145
17th " "	170	165
18th " "	165	170
19th " "	140	140

MISCELLANEOUS WRITTEN EXERCISES

1. Construct on the same chart the weekly sales of four salesmen from the following data:

SALESMEN	1ST WK.	2D WK.	3D WK.	4TH WK.	5TH WK.	6TH WK.	7TH WK.	8TH WK.
No. 1....	\$92	\$ 30	\$176	\$150	\$105	\$110	\$ 80	\$ 88
" 2....	56	58	86	88	140	95	98	76
" 3....	60	116	120	150	245	210	76	220
" 4....	76	380	230	290	310	270	240	325

The above-mentioned sales were actually made by different men selling a house-to-house article on a commission basis. The curves will immediately show up the comparison of their sales.

2. The following data shows that an automobile should stop within the given distances according to the miles per hour of speed, if the brakes are in proper order. Construct a curve from this data.

AT SPEED OF	A CAR SHOULD STOP IN
10 miles per hr.....	9.2 ft.
15 " " "	20.8 "
20 " " "	37 "
25 " " "	58 "
30 " " "	83.3 "
35 " " "	104 "
40 " " "	148 "
50 " " "	231 "

3. The following information shows the maximum percentages of different family annual incomes which ought to be expended in normal times for the variously named parts of a family expense account.

VARIOUS ITEMS	PERCENTAGE OF ANNUAL INCOME			
	\$2,000-\$4,000	\$1,000-\$2,000	\$800-\$1,000	\$500-\$800
Food.....	25	25	30	45
Rent.....	20	20	20	15
Clothing.....	15	20	15	10
Miscellaneous operating..	8	5	5	7
Higher living, books, savings, insurance, religious, etc.....	24	21	18	11
Railroad.....	2	2	2	2
Street-car.....	1	1	3	5
Water rent.....	1	1	1	1
Gas.....	2	3	5	4
Electricity.....	1	1	1	
Telephone.....	1	1		100
	100	100	100	

Construct a circle or a rectangle for each income and divide it up into its component parts. Then place the circles or rectangles side by side to show comparisons.

4. The following facts taken from the *New York Times Annualist* show the trend of bond prices over a certain period of time. Plot a curve from the following data:

1917	PRICE	1918	PRICE
Jan.....	90	Jan.....	77
Feb.....	87	Feb.....	76 $\frac{1}{2}$
Mar.....	86 $\frac{1}{2}$	Mar.....	76
Apr.....	85	Apr.....	76 $\frac{1}{4}$
May.....	83	May.....	77 $\frac{1}{2}$
June.....	84	June.....	76 $\frac{1}{2}$
July.....	82 $\frac{1}{2}$	July.....	76 $\frac{3}{4}$
Aug.....	83	Aug.....	76 $\frac{1}{2}$
Sept.....	80	Sept.....	76
Oct.....	79 $\frac{1}{2}$	Oct.....	77 $\frac{3}{4}$
Nov.....	77	Nov.....	82
Dec.....	74 $\frac{1}{4}$	Dec.....	80

5. Each dollar of cash income of the New York Life Insurance Company, was expended as follows according to a recent yearly report of the company. Construct a graph to represent these facts.

Paid for death claims.....	\$.21
Paid to living policyholders.....	.38
Set aside for reserve and dividends.....	.29
Paid to agents.....	.06
For branch expenses, agency supervision, and medical inspection.....	.02
For administration and investment expenses.....	.03
For insurance, debt, taxes, fees, and licenses.....	.01
Total.....	\$1.00

6. Construct three curves on the same chart (preferably with different colored inks) to show the changes in the population of New York, Chicago, and Pittsburgh respectively, from the following data:

YEAR	POPULATION OF NEW YORK	POPULATION OF CHICAGO	POPULATION OF PITTSBURGH
1850.....	480,000	100,000	20,000
1860.....	325,000	200,000	80,000
1870.....	425,000	210,000	90,000
1900.....	3,437,202	1,698,575	451,512
1910.....	4,766,883	2,185,283	533,905
1920.....	5,620,048	2,701,705	588,193

7. The United States life tables, Census 1910, report the following death rates per 1,000 among the white males at the various ages. Construct the curve from the data and compare the numbers of different ages.

AGE	DEATH RATE PER 1,000	AGE	DEATH RATE PER 1,000
12	2.4	70.....	69
20.....	4.5	80.....	137
30.....	6.8	90.....	245
40.....	12	100.....	386
50.....	18	106.....	585
60.....	36		

8. Refer to §138 and construct a chart showing the following information from averaged figures, taken from many trade organizations, the Harvard Bureau of Business, and individual investigations.

PERCENTAGE OF TOTAL SALES

KIND OF BUSINESS	NET PROFITS	CASH DISCOUNTS	COST OF DOING BUSINESS
Variety goods.....	6	3	19
Dry goods.....	5	4	23
Clothing.....	5.25	5.5	23.5
Furniture.....	7.75	3.25	23.75
Jewelry.....	3	6.5	25
Drugs.....	5	3	25
Hardware.....	7.75	6.25	19.5
Shoes.....	4	3	25
Department stores.....	6	0	25
Implements and vehicles	2.25	6.5	17.5
Groceries.....	2	2	17

9. Refer to §138 and construct a chart from the following information of profits, costs, and discounts by lines—net profits for one turnover.

PERCENTAGE OF TOTAL SALES

LINE	NET PROFITS	CASH DISCOUNTS	COST OF DOING BUSINESS
Books.....	2	3	22
Corsets.....	8	4	24
Furs.....	7.5	3.25	26
Gloves.....	4	5	24
Hosiery.....	5	5.5	23.5
Handkerchiefs.....	4	3.5	24.5
Laces.....	9.5	3.25	23.25
Linens.....	6	3	24
Millinery.....	10	6	25

PERCENTAGE OF TOTAL SALES

LINE	NET PROFITS	CASH DISCOUNTS	COST OF DOING BUSINESS
Pictures.....	13	4	25
Ribbons.....	3.5	4	24
Silks.....	4	5	23
Toys.....	10.75	3.25	22
Umbrellas.....	5	4	26
Wash Goods.....	7	3.5	20.5

10. Referring to the tables in §§ 18 and 21, which show the corresponding profit on cost represented by certain per cents profit on selling price, solve one of the above examples for profits on cost and note comparison of your answers with those enumerated above.

11. Make a comprehensive chart from the following facts, somewhat after the idea of § 138, with these suggestions: Use a scale on the left for capitalization up to 145 millions; one on the right up to 25 millions; but the right-hand scale to be 4 times the left-hand scale, i.e., 20 millions on the left is the same line as 5 millions on the right. The lower right-hand scale is to represent earnings. In the upper right-half of the chart have a scale for price range of the company's general and refunding 5% bonds. Let 75 on this scale be on a line with 100 on the left, and 80 shall correspond with 108 on the left, etc. Put in the price range curve with red.

YEAR	IN MILLIONS OF DOLLARS								DOLLARS
	CAPITALIZATION	COMMON STOCK	PREFERRED STOCK	BONDS	OPERATING EXPENSES & TAXES	INTEREST	BALANCE	GROSS EARNINGS	RANGE OF PRICES OF BONDS
1909.....	87	20	10	57	31	11	10	52	
1910.....	88	20	10	58	33	11	11	55	
1911.....	103	28	10	65	34	11	12	57	\$92
1912.....	117	30	10	77	34.7	11.3	11	57	89
1913.....	122	32	10	80	40.5	11.5	12	64	83
1914.....	130	32	19	79	43	12	13	68	88
1915.....	132	34	22	76	42	15	18	75	90
1916.....	135	35	23	77	43	15	18	76	93
1917.....	138	36	23	79	50	15	14	81	77
1918.....	141	36	23	82	60	15	15	90	88

12. From the following data, taken from Paul Nystrom's book, "The Economics of Retailing," make ten circles of equal size or ten rectangles of equal length, showing the component parts for the apportionment of the rent in 10 stores of different numbers of floors. Study and compare your results.

	PERCENTAGE OF RENT									
	1	2	3	4	5	6	7	8	9	10
Basement.....	35		25		15	10	10	15	12.5	15
Main floor.....	65	65	50	60	45	45	50	40	35	
Second floor.....		35	25	30	25	25	20	20	20	20
Third ".....				10	15	10	10	15	15	10
Fourth ".....						10	10	10	10	10
Fifth ".....									7.5	5
Sixth ".....										5
First ".....										35

13. Construct two curves to represent the time it takes two operators to do various operations. Add the time for each operation to the total time spent on the preceding operations, thus forming a cumulative time for the whole number of operations

KINDS OF OPERATIONS	TIME IN SECONDS	
	FIRST OPERATOR	SECOND OPERATOR
Reaches for label.....	2	3
Reaches for brush.....	3	3
Wipes brush on glue pot.....	6	9
Brings brush to label.....	2	3
Covers label with glue.....	5	9
Replaces brush.....	2	2
Puts label on package.....	3	4
Adjusts and smooths label.....	6	12

14. Construct three curves on the same chart field to show the percentage of the pupils of each teacher's class which attained the various values (or percentages) in the same grade of work.

VALUES	NUMBER OF STUDENTS	PER CENT OF TOTAL NUMBER IN CLASS
First Teacher		
40- 50	2	5
50- 60	2	5
60- 70	4	10
70- 80	20	50
80- 90	10	25
90-100	2	5

VALUES NUMBER OF STUDENTS PER CENT OF
TOTAL NUMBER IN CLASS

Second Teacher

50- 60	2	5
60- 70	2	5
70- 80	15	37.5
80- 90	10	25
90-100	11	27.5

Third Teacher

50- 70	0	0
70- 80	20	50
80- 90	10	25
90-100	10	25

15. Construct a chart to show the comparison of the expenditure of \$1 in 2 successive years, of the railroads of the United States, from the following data:

	IN CENTS	
	1ST YR.	2D YR.
Operating expenses.....	68.1	62.55
Taxes.....	7.03	4.25
Excess of fixed charges over non-operat- ing income.....	10.13	11.14
Dividends.....	14.74	22.06

16. Construct from the following information three curves on each of three charts of the same height and scale, to show how a certain business is running.

	JAN.	FEB.	MAR.	APR.	MAY	JUNE
1st Chart						
Receipts.....	\$1,375	\$1,350	\$1,450	\$1,425	\$1,400	\$1,350
Purchases.....	925	850	1,100	1,000	950	1,000
Total expense.....	600	525	520	550	500	475
2d Chart						
Total expense.....	625	550	525	500	550	525
Total salary expenses..	440	420	450	420	410	420
Personal.....	340	320	310	320	340	320
3d Chart						
Gross profit.....	38	43	31	38	35	40
Expense.....	35	25	25	26	25	28
Net profit.....	3	18	6	12	10	12

17. Chart the following facts by the use of the same-sized circles or parts of a circle to show the dollar's buying power and its changes during a part of the World War.

	% BUYING POWER
1st yr.....	100
2d "	90.6
3d "	84.28
4th "	58.8

18. Construct two rectangles of the same length and divide each into its component parts to show how rents vary in different lines. These figures were compiled by the Bureau of Business Research of Harvard University. Note how the rents of the two respective lines vary.

	RATIO BETWEEN RENT AND NET SALES		
	HIGH	LOW	COMMON OR TYPICAL
Groceries.....	4.1%	.3%	1.3%
Shoes.....	14.6%	.8%	5%

19. Chart three curves on the same field to show the following facts concerning the railroads of the United States for the year. These facts are from the Bureau of Railroad Economics, Washington, D. C.

	JAN.	FEB.	MAR.	APR.	MAY	JUNE
Revenues.....	\$1,125	\$1,150	\$1,250	\$1,225	\$1,300	\$1,300
Expenses.....	800	800	850	825	860	850
Net revenues.....	325	350	400	400	440	450

	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Revenues.....	\$1,320	\$1,410	\$1,400	\$1,470	\$1,400	\$1,350
Expenses.....	850	890	890	900	890	900
Net revenues.....	470	520	510	570	510	450

20. A man walks to a place at the rate of 4 miles per hour, remains 1 hr., then rides back at the rate of 10 miles per hour. He was absent 8 hr. How far did he walk?³

³ From H. E. Cobb, Elements of Applied Mathematics. Boston, Ginn and Company, 1911.

HINT: To graph, or solve graphically, take 8 spaces from the intersection of the vertical and horizontal axes (called 0) on the horizontal. Call it P. Let the horizontal scale represent hours, and the vertical scale miles. Draw a line from 0 through X the intersection of the lines representing 1 hr. and 4 miles, to the opposite side of the chart. Do the same from P toward the left 1 hr. and up 10 miles to a point Y. Draw line from P through Y until it crosses the former line. Find the points in the two lines on a horizontal, which are 1 hr. apart. Follow that line to the left and you will find the number of miles.

21. A man rows at the rate of 6 miles per hr. down stream to a place, and 2 miles per hr. in returning. How many miles distant was the place if he was absent 12 hr. and remained at the place 6 hr.?

22. The expenses of a firm were reported as follows for a certain year. Graph these facts so that you could present them in good form to the president of the company.

Postage.....	\$ 1,000	Advertising.....	\$17,000
Telephone.....	2,000	Equipment.....	1,400
Selling.....	17,000	Managing.....	6,000

23. Try to find out the main branches of the Ford Motor Company, their location, etc., and make a pin map of the branches that are in the United States.

24. Find out the branch offices of some large corporation or manufacturing plant in your locality and then construct a map, and on it mark these branch offices.

25. Apply Exercise 23 to Swift and Company, Chicago, Ill.

26. Try to obtain similar information from the United Cigar Stores Company of New York and make a pin map.

27. Obtain similar information concerning the Woolworth 5 and 10 cent stores and make a pin map.

28. Find out the location of the branches of the United States Steel Corporation and apply Exercise 23.

29. Do the same for the Standard Oil Company.

CHAPTER XII

SHORT METHODS AND CHECKS

143. Value of Short-Cut and Checking Methods.—A computer naturally should be the master of short-cuts and methods for checking his work. The latter is perhaps of more importance than the former, because one must be able to show his employer that he knows that his work is correct.

The executive should also know these methods in order that he may be able to check the work of his employees. He must also be sure of the facts presented as a basis for a satisfactory and successful business.

It is the object of this chapter to give to any person who will devote a small amount of time to it each day, such a knowledge that he will be well equipped to do computation work in the shortest as well as the best way, and at the same time be able to make an accurate check on these computations.

144. Addition.—"Anything worth doing is worth doing well," therefore we should not be satisfied with a piece of work unless we check it in some way, if it is at all possible, and with most work it is possible. The checks that are quite common in addition are:

1. Adding the example from top to bottom, then adding from bottom to top, or vice versa.
2. Adding by columns.

Illustrative Example.	(a)		(b)
	1456		1456
	7238		7238
	3564	or	3564
	18		11
	14		11
	11		14
	11		18
	12258		12258

EXPLANATION: (a) Add each column separately, beginning at the right, and set the result for each column down by itself. The result for the tens column should be placed one place to the left of that for the units column. Then add the totals. This method is valuable when adding long columns, especially if one is interrupted while adding.

(b) Add in a similar manner beginning at the left column and moving each result one place to the right.

3. Excesses of 9's, often called "casting out 9's."

Illustrative Example.	1457	8
	7238	2
	3564	0
	12259	1
	1	↙

EXPLANATION: Add the digits (or figures) in each number (as for example in 1,457), $1 + 4 + 5 + 7 = 17$; divide 17 by 9 and the amount remaining, 8, is placed out at the right. Do the same for all the numbers. Then add these excesses (or amounts left over) and the result is 10 in the above example; find the excesses of 9 in this sum, giving 1 in the above. Then find the excesses of 9 in the total, 12,259, which we also find is 1.

This gives a fairly satisfactory check. This check, however, is not always to be depended upon because one may make a mistake of 9 (or some multiple of 9) or 0 in the addition which will not show as an error in the excesses of 9's in the total or a mistake in arranging the figures in a number, i.e., 4,175 instead 1457, above.

WRITTEN EXERCISES

Try out these checks on the following examples:

1. 4,567	2. 6,597	3. \$1,467.75
3,784	5,836	5,693.83
2,543	6,784	7,649.75
6,738	5,987	6,573.81
	6,774	5,967.48
	5,965	6,796.54

HINT: In all addition try finding numbers that group and make 10, or 5, or something similar.

$$\begin{array}{r} \text{Example: } 7 \\ 3 \end{array} \} 10$$

$$\begin{array}{r} 2 \\ 4 \end{array} \} 6$$

145. Subtraction.—The common check is the process opposite to the usual one, i.e., add the remainder to the subtrahend and it will produce the minuend if the work is correct.

Illustrative Example.

$$\begin{array}{r} 567 \\ 198 \\ \hline 369 \end{array} \quad \text{CHECK: } 369 + 198 = 567$$

WRITTEN EXERCISES

Find the difference in the following examples and check:

1. 145.674	2. 7,564	3. 4,563
96.785	3,783	7,862

Another check is the subtraction of the excesses of 9's.

Illustrative Example.

$$\begin{array}{r}
 196 \quad 7 \\
 157 \quad 4 \\
 \hline
 39 \quad 3 \\
 3 \quad \swarrow
 \end{array}$$

WRITTEN EXERCISE

1. Add columns (a) and (c) and subtract column (b) from the result for column (d)

(a)	(b)	(c)	(d)
BALANCE AT BEGINNING	CHECKS	DEPOSITS	BALANCE
\$150	\$ 75	\$125	
215	205	137	
316	187	49	
149	258	325	
256	385	234	
—	274	425	
324	—	124	

How would you check this problem? Check it.

Another check is to subtract by addition or to subtract a sum of two or more numbers from a certain number.

Illustrative Example. 14563

$$\begin{array}{r}
 14563 \\
 3754 \\
 2649 \\
 5365 \\
 \hline
 2795
 \end{array}$$

EXPLANATION: Adding from the bottom up in the first column, 5, 14, 18, and what make 23, put the result, 5, down and carry 2; then the second column 2, 8, 12, 17, and what make 26. Set down the 9 and carry the 2 to the third column; 2, 5, 11, 18, and what make 25, answer 7. Set down the 7 and carry 2 to the last column; 2, 7, 9, 12, and what make 14. Set down the answer 2. Now check the work by adding up all the numbers but the top one, and the result should be the top number.

WRITTEN EXERCISES

Try this plan on the following examples and find the balances in the bank at the end of the day.

1. \$2345 balance in the bank in the morning

\$564	} Checks given out during the day
243	
367	
568	

BALANCE	CHECKS	LAST BALANCE
2. \$ 567.85	\$124.65; \$ 56.76; \$75.64	
1,256.65	546.73; 124.75; 75.75	

146. Multiplication.—Methods of checking multiplication are as follows:

1. Divide the product by the multiplier to obtain the multiplicand, or by the multiplicand to obtain the multiplier.

2. Repeat the multiplication and if the result is as before we may assume that the work is correct.

3. Cast out 9's, as follows:

Illustrative Example. 43×21

43	7 excess
21	3 "
—	—
903	21
3 =	3

EXPLANATION: Find the excesses of 9's in the multiplier and the multiplicand. Multiply these excesses together and find the excess of 9's in this product.

WRITTEN EXERCISES

Multiply the following and check:

1. 256	2. 345	3. 456	4. 375	5. 451
25	31	123	256	231
—	—	—	—	—

By the use of the excesses of 9's, without actually multiplying out, state which of the following results are correct:

6.	456	7.	376	8.	415	9.	575	10.	56
	25		75		16		125		22
	11,400		28,200		6,440		71,875		1,232

Short methods prove to be very valuable. Thoroughly understand each method as you go along and practice it whenever possible.

147. To Multiply by Any Multiple of 10.—Move the decimal point as many places to the right as there are zeros in the multiplier. It is obviously necessary to annex zeros if the multiplicand is a whole number as a decimal point is understood at the end of each whole number.

Illustrative Example 1. $.456 \times 100$

SOLUTION: Move the decimal point two places to the right, giving 45.6

Illustrative Example 2. $1.347 \times 10,000$

SOLUTION: Move the decimal point four places to the right, giving 13,470.

Illustrative Example 3. $15 \times 1,000$

SOLUTION: Since there is a decimal point understood at the right of any whole number, therefore in this case we move it three places to the right, giving 15,000.

ORAL EXERCISES

Multiply the following mentally:

- | | | | | | |
|-----------|-----------------|-------|------------|------------------|--------|
| 1. | 456 $\times$ | 10 | 6. | 2.467 $\times$ | 10,000 |
| 2. | 64.7 $\times$ | 100 | 7. | .00035 $\times$ | 10,000 |
| 3. | 456.25 $\times$ | 1,000 | 8. | 4.00016 $\times$ | 1,000 |
| 4. | 6.413 $\times$ | 100 | 9. | 546.1 $\times$ | 1,000 |
| 5. | .00005 $\times$ | 1,000 | 10. | 310 $\times$ | 10,000 |

148. To Multiply Numbers Having Zeros as Endings.—Multiply by the significant figures, then annex as many zeros as there are in both the multiplicand and the multiplier.

Illustrative Example 1. 430×400

SOLUTION: Multiply 43 by 4, giving 172, then annex three zeros, giving 172,000.

ORAL EXERCISES

Multiply the following:

- | | |
|-------------------------|--------------------------|
| 1. 236×20 | 6. $1,350 \times 900$ |
| 2. 352×300 | 7. $18,000 \times 12$ |
| 3. $456 \times 1,200$ | 8. $140 \times 2,500$ |
| 4. $2,145 \times 2,200$ | 9. $230 \times 4,000$ |
| 5. 120×400 | 10. $1,600 \times 1,200$ |

Illustrative Example 2. $.256 \times 20$

SOLUTION:

$$.256 \times 10 = 2.56$$

$$2.56 \times 2 = 5.12$$

or

$$.256 \times 2 = .512$$

$$.512 \times 10 = 5.12$$

Multiply the following:

- | |
|--------------------------|
| 1. $.53 \times 300$ |
| 2. 16.351×400 |
| 3. $.056 \times 120$ |
| 4. $.0027 \times 9,000$ |
| 5. $2.136 \times 30,000$ |

149. To Multiply by 9, 99, 999, etc.—To multiply a number by 9, multiply the number by 10, and then subtract the original number from the result of the multiplication.

Illustrative Example. 456×9

SOLUTION:

$$\begin{array}{r} 4560 = 10 \times 456 \\ 456 \\ \hline 4,104 \end{array}$$

To multiply a number by 99, multiply the number by 100, and subtract the number.

Illustrative Example. 345×99

SOLUTION:

$$\begin{array}{r} 34500 = 100 \times 345 \\ 345 \\ \hline 34155 \end{array}$$

WRITTEN EXERCISES

Multiply each of the following numbers by 9, 99, and 999:

- | | |
|--------|--------|
| 1. 238 | 4. 642 |
| 2. 426 | 5. 233 |
| 3. 175 | |

150. To Multiply by 25, 50, 75, etc.—Since 25 is equal to $\frac{100}{4}$ therefore to multiply a number by 25, multiply the number by 100, and divide the result by 4.

Illustrative Example 1. 348×25

SOLUTION:

$$\begin{array}{r} 4 \overline{) 34,800} \\ 8,700 \end{array}$$

Since $75 = 100 - 25 = 100 - (\frac{1}{4} \text{ of } 100)$, therefore to multiply a number by 75, multiply the number by 100, divide by 4, and subtract.

Illustrative Example 2. 576×75

SOLUTION:

$$\begin{array}{r} 4 \overline{) 57,600} \\ \underline{14,400} \\ 43,200 \end{array}$$

WRITTEN EXERCISES

1. Multiply 679 by 50. How would you do it?

2. How would you multiply by 125?

Find the results of the following by the above methods:

- | | | |
|-------------------|----------------------|------------------------|
| 3. 32×25 | 7. 67×50 | 11. $1,248 \times 125$ |
| 4. 76×25 | 8. 88×75 | 12. $3,457 \times 25$ |
| 5. 84×25 | 9. 145×50 | 13. $1,256 \times 750$ |
| 6. 43×50 | 10. 726×250 | 14. 496×250 |

151. To Multiply Two Numbers, Each Ending in 5.—

Illustrative Example 1. $65 \times 65 = 4,225$

EXPLANATION: When the same numbers are to be multiplied, write 25 for the last two figures at the right. Then add 1 to the tens figure ($6 + 1 = 7$) and multiply by the other tens figure ($7 \times 6 = 42$), and write this result at the left of the 25 just written. The product is 4,225.

Illustrative Example 2. When the sum of the tens figures is an even number. $55 \times 75 = 4,125$.

EXPLANATION: Set down the 25 as the first part of the product. Then find $\frac{1}{2}$ of the sum of the tens figures ($\frac{1}{2}$ of 12 = 6), and add it to the product of the tens figures. $5 \times 7 = 35$. $35 + 6 = 41$. Place the 41 at the left of the 25 previously written.

Illustrative Example 3. When the sum of the tens figures is an odd number. $35 \times 25 = 875$

EXPLANATION: Set down 75 as the first part of the product. Then find $\frac{1}{2}$ of the sum of the tens figures ($\frac{1}{2}$ of 5 = $2\frac{1}{2}$). Drop the $\frac{1}{2}$, and add the 2 to the product of the tens figures.

ORAL EXERCISES

Multiply the following:

- | | |
|-------------------|-------------------|
| 1. 35×35 | 6. 65×15 |
| 2. 85×85 | 7. 35×45 |
| 3. 15×15 | 8. 45×55 |
| 4. 65×25 | 9. 125^2 |
| 5. 85×45 | 10. 75^2 |

152. To Square Any Number of Two Figures.—This is based upon the principle that the square of the sum of two quantities, like $(a + b)^2$, is equal to the square of the first quantity, plus twice the first quantity times the second quantity, plus the square of the second quantity.

Illustrative Example.

$$\begin{aligned}(3 + 4)^2 &= 3^2 + 24 + 4^2 \\ &= 49\end{aligned}$$

WRITTEN EXERCISES

Square the following numbers by the above method:

- | | |
|-------|---------------------------|
| 1. 64 | 4. 52 |
| 2. 36 | 5. 124 (Call it 12 tens.) |
| 3. 27 | |

153. Sum of Two Quantities Times Their Difference.—Certain number products come under the principle that the sum of two quantities times the difference of those same two products, as $(a + b)(a - b)$ equals the square of the first quantity minus the square of the second quantity, or $a^2 - b^2$.

Illustrative Example.

$$\begin{aligned}21 \times 19 &= (20 + 1)(20 - 1) \\ &= 400 - 1\end{aligned}$$

WRITTEN EXERCISES

Find the product of the following numbers by the above method:

1. 22×18

4. 64×56

2. 36×44

5. 62×58

3. 53×47

154. To Multiply by 11, 22, or Any Multiple of 11.—

Illustrative Example 1. 264×11

$$\begin{array}{r} \text{SOLUTION:} \quad 264 \\ \quad \quad 11 \\ \hline 2,904 \end{array}$$

EXPLANATION: Set down the units figure of the given number in the product, 4. Add the units and tens figures ($6 + 4 = 10$). Set down the zero and carry 1. Add the tens and hundreds figures ($6 + 2 = 8$); add the 1 carried, making 9. Multiply the last figure (hundreds in this example) by 1, set the result, 2, down. Result = 2,904.

Illustrative Example 2. 416×22

$$\begin{array}{r} \text{SOLUTION:} \quad 416 \\ \quad \quad 22 \\ \hline 9,152 \end{array}$$

EXPLANATION: Two times the unit figure = 12. Set down the 2 and carry 1. Add the units and tens figures ($6 + 1 = 7$). Multiply the 7 by 2 = 14, and 1 carried makes 15. Set down the 5 and carry 1. Add the tens and hundreds figures ($1 + 4 = 5$). Multiply the 5 by 2 and add the 1 carried, making 11. Set down the 1 and carry 1. Multiply the last figure (4 in this example) by 2 and add the 1 carried, making 9. Result = 9,152.

WRITTEN EXERCISES

Multiply the following:

1. 346×11

5. $1,246 \times 22$

2. 562×22

6. 322×44

3. 124×22

7. $1,416 \times 88$

4. 214×33

8. $3,514 \times 77$

155. To Multiply by a Number Composed of Factors.—
If certain figures of the multiplier are factors of the other figures in the multiplier, the following method can be used:

Illustrative Example. 356×568

$$\begin{array}{r}
 \text{SOLUTION:} \quad 356 \\
 \quad \quad \quad 568 \\
 \hline
 \quad \quad \quad 2848 = 8 \times 356 \\
 \quad 19936 = 7 \times 2848 \\
 \hline
 \quad 202,208
 \end{array}$$

HINT: Be careful about placing the result of the second operation.

WRITTEN EXERCISES

Multiply the following according to the method illustrated above:

- | | |
|----------------------------|------------------------------|
| 1. 624×426 | 3. 126×124 |
| 2. 358×244 | 4. $1,718 \times 186$ |

156. To Multiply Two Numbers between 10 and 19 Inclusive.—

Illustrative Example. 16×14

$$\begin{array}{r}
 \text{SOLUTION:} \quad 200 \\
 \quad \quad \quad 24 \\
 \hline
 \quad \quad 224
 \end{array}$$

EXPLANATION: To either number add the units figure of the other number and annex 0, then add the product of the units figures.

Multiply the following according to the method explained above:

- | | | | |
|----------------|----------------|----------------|----------------|
| 1. 17 | 3. 18 | 5. 17 | 7. 19 |
| 18 | 13 | 15 | 13 |
| <hr/> | <hr/> | <hr/> | <hr/> |
| 2. 16 | 4. 19 | 6. 18 | 8. 14 |
| 15 | 17 | 16 | 18 |

157. To Multiply by Use of Aliquot Parts of 10.-**Illustrative Example 1.** $28 \times 1\frac{1}{4}$ **SOLUTION:** $280 \div 8 = 35$ **EXPLANATION:** Since $1\frac{1}{4} = \frac{10}{8}$, therefore to multiply by $1\frac{1}{4}$, multiply the number by 10, and divide by 8.**Illustrative Example 2.** $72 \times 6\frac{2}{3}$ **SOLUTION:** $3 \overline{) 720}$

240

480

$$6\frac{2}{3} = 10 - 3\frac{1}{3} = 10 - (\frac{1}{3} \text{ of } 10)$$

 $\therefore$ Multiply by 10, divide by 3, and subtract.**WRITTEN EXERCISES****1.** How could you plan to multiply by $7\frac{1}{2}$?

Find the product of the following by the above method:

2. $124 \times 2\frac{1}{2}$

7. $7,564 \times 2\frac{1}{2}$

3. $1,728 \times 3\frac{1}{3}$

8. $15,624 \times 1\frac{1}{4}$

4. $144 \times 6\frac{2}{3}$

9. $356 \times 7\frac{1}{2}$

5. $256 \times 7\frac{1}{2}$

10. $1,236 \times 6\frac{2}{3}$

6. $1,456 \times 1\frac{1}{4}$

11. $3,567 \times 3\frac{1}{3}$

158. To Multiply by Use of Aliquot Parts of 100.—**Illustrative Example 1.** 624×25 **SOLUTION:** $62,400 \div 4 = 15,600$ **EXPLANATION:** Since $25 = \frac{100}{4}$, therefore multiply by 100, and divide by 4.**Illustrative Example 2.** 456×75 **SOLUTION:** $4 \overline{) 45,600}$

11,400

34,200

EXPLANATION: Since $75 = 100 - 25 = 100 - (\frac{1}{4} \text{ of } 100)$

WRITTEN EXERCISES

Find the total value of each of the following:

1.	2.	3.
25 yd. at 44¢	25 lb. at 45¢	12½ yd. at 72¢
33⅓ " " 36¢	37½ " " 72¢	8⅓ " " 24¢
12½ " " 48¢	31¼ " " 32¢	33⅓ " " 14¢
16⅔ " " 60¢	16⅔ " " 42¢	62½ " " 24¢
4.	5.	6.
60 lb. at 16⅔¢	120 yd. at 62½¢	16 articles at \$12½
90 " " 6⅔¢	300 " " 83⅓¢	12 " " \$1¼
48 " " 6¼¢	32 articles at \$25	28 " " \$125
56 " " 37½¢	996 bu. at \$62½	18 " " \$1⅔

159. To Divide by 10, 100, 1,000, etc.—

Illustrative Example. $4,675 \div 10 = 467.5$

EXPLANATION: Move the decimal point as many places to the left as there are zeros in the divisor.

ORAL EXERCISES

Divide each of these numbers by 10, 100, 1,000:

1. 14,500.	5. .124	9. 67.346
2. 675.64	6. .000643	10. 7,865.4
3. 567.52	7. 12 36	11. 649
4. 695.6	8. 724.74	12. 7,567.4

160. To Divide by 25, 50, 75, etc.—

Illustrative Example 1. $12,400 \div 25 = 124 \times 4 = 496$

EXPLANATION: Since $25 = \frac{100}{4}$, therefore to divide by 25, divide by 100, and multiply by 4.

Illustrative Example 2. $3,000 \div 75$

$$\begin{aligned} 3,00 &\div 100 = 30 \\ 30 &+ (\frac{1}{3} \text{ of } 30) = 40 \end{aligned}$$

WRITTEN EXERCISES

Perform each of the following divisions:

- | | |
|-------------------------------|--------------------------------|
| 1. $6,400 \div 25$ | 6. $13,450 \div 25$ |
| 2. $3,300 \div 33\frac{1}{3}$ | 7. $46.755 \div 33\frac{1}{3}$ |
| 3. $4,600 \div 50$ | 8. $143.154 \div 75$ |
| 4. $12,300 \div 75$ | 9. $56.9 \div 50$ |
| 5. $2,700 \div 50$ | 10. $.0145 \div 25$ |

161. To Divide by $2\frac{1}{2}$, $3\frac{1}{3}$, etc.—

Illustrative Example 1. $50 \div 2\frac{1}{2}$

SOLUTION:

$$2\frac{1}{2} = \frac{1}{4} \text{ of } 10$$

$$50 \div 10 = 5$$

$$4 \times 5 = 20$$

Illustrative Example 2. $80 \div 6\frac{2}{3}$

SOLUTION:

$$10 - 6\frac{2}{3} = 3\frac{1}{3}$$

$$3\frac{1}{3} = \frac{1}{2} \text{ of } 6\frac{2}{3}$$

$$80 \div 10 = 8$$

$$\frac{1}{2} \text{ of } 8 = 4$$

$$8 + 4 = 12$$

WRITTEN EXERCISES

Perform the following divisions:

- | | | |
|----------------------------|-----------------------------|----------------------------|
| 1. $120 \div 2\frac{1}{2}$ | 4. $60 \div 6\frac{2}{3}$ | 7. $360 \div 1\frac{1}{4}$ |
| 2. $150 \div 3\frac{1}{3}$ | 5. $.624 \div 5$ | 8. $540 \div 7\frac{1}{2}$ |
| 3. $240 \div 7\frac{1}{2}$ | 6. $31.6 \div 2\frac{1}{2}$ | |

162. To Divide by a Number Composed of Factors.—

Illustrative Example. $2,352 \div 42$

SOLUTION:

$$\begin{array}{r} 7 \overline{) 2,352} \\ 6 \overline{) 336} \\ \underline{56} \end{array}$$

EXPLANATION: $42 = 6 \times 7$

163. To Divide by Continued Subtraction.—This method has been used as a catch question.

Illustrative Example. $645 \div 147$

SOLUTION: 645

(1) 147

498

(2) 147

351

(3) 147

204

(4) 147 Result = 4 (number of subtractions are 4)

$57 =$ remainder

164. Checks in Division.—1. Multiply the quotient by the divisor, and add the remainder.

2. Cast out 9's, as follows:

Illustrative Example. $7,564 \div 143 = 52$, with a remainder of 128.

SOLUTION:

Excesses of 9's in $7,564 = 4$

" " " " $143 = 8$

" " " " $52 = 7$

" " " " $128 = 2$

$8 \times 7 = 56$

Excesses of 9's in $56 = 2$

$2 + 2 = 4$

$\therefore$ The excesses in the dividend = the excesses in the remainder + the excesses in the product of the excesses of the divisor and the quotient.

WRITTEN EXERCISES

Divide the following and check:

1. $7,563 \div 246$

4. $6,574 \div 925$

2. $13,674 \div 345$

5. $76,348 \div 346$

3. $5,692 \div 74$

165. Addition of Numbers Containing Fractions.—

Illustrative Example 1. $\frac{3}{4} + \frac{4}{5} + \frac{2}{3} + \frac{5}{6} = ?$ **SOLUTION:** $\frac{3}{4}$ 45 $\frac{4}{5}$ 48 $\frac{2}{3}$ 40 $\frac{5}{6}$ 50

183

= $3\frac{1}{2}$

60

Write the least common denominator only at the end of the work.

Illustrative Example 2. $15\frac{1}{6} + 14\frac{2}{3} + 65\frac{3}{4} = ?$ **SOLUTION:** $15\frac{1}{6}$ 2 $14\frac{2}{3}$ 8 $65\frac{3}{4}$ 9

 $94 + \frac{19}{12} = 95\frac{7}{12}$ **Illustrative Example 3.** How many yards in 4 pieces of cloth containing 21^2 , 36^1 , 43^3 , 56^1 ?**SOLUTION:** 21^2 36^1 43^3 56^1

 157^3

(The small numbers at the right and above represent quarter yards.)

Illustrative Example 4. $\frac{1}{6} + \frac{1}{5} = ?$ Ans. $\frac{11}{30}$ **NOTE:** It will be noted that the numerator is the sum of the denominators, and the denominator is the product of the denominators.**Illustrative Example 5.** $\frac{2}{6} + \frac{2}{3} = \frac{22}{30}$ or $\frac{11}{15}$ **NOTE:** Observe that the numerator is 2 times what it was in Example 4, and the denominator the same.**Illustrative Example 6.** To add fractions by cross-multiplying the denominators and numerators. $\frac{5}{6} + \frac{3}{7} = ?$ **SOLUTION:** $5 \times 7 = 35$ $3 \times 6 = 18$ $35 + 18 = 53$, the new numerator $7 \times 6 = 42$, the new denominator $\frac{53}{42} = \text{result}$

166. Subtraction of Fractions.—

Illustrative Example. When the numerators are alike. $\frac{3}{4} - \frac{3}{5}$

SOLUTION: $5 \times 3 = 15$
 $4 \times 3 = 12$
 $15 - 12 = 3$, the new numerator
 $4 \times 5 = 20$, the new denominator
 $\frac{3}{20} = \text{result}$

167. Multiplication of Mixed Numbers.—

Illustrative Example. $14\frac{1}{2} \times 8\frac{2}{3}$

SOLUTION: $14\frac{1}{2}$
 $8\frac{2}{3}$

 $\frac{1}{3} = \frac{1}{2} \times \frac{2}{3}$
 $4 = 8 \times \frac{1}{2}$
 $9\frac{1}{3} = \frac{2}{3} \times 14$
 $112 = 8 \times 14$

 $125\frac{2}{3} = \text{result}$

168. Multiplication of Mixed Numbers Whose Fractions Are $\frac{1}{2}$.—

Illustrative Example 1. $8\frac{1}{2} \times 8\frac{1}{2} = ?$

SOLUTION:

$8\frac{1}{2}$
 $8\frac{1}{2}$

 $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$
 $8 = \frac{1}{2} \text{ of } 8 + \frac{1}{2} \text{ of the other } 8, \text{ or } \frac{1}{2} \text{ of } (8 + 8)$
 $64 = 8 \times 8$

 $72\frac{1}{4} = \text{result}$

or

$$8\frac{1}{2}$$

$$8\frac{1}{2}$$

$$\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$$

$$72 = (8 + 1) \text{ times the other } 8$$

$$72\frac{1}{4} = \text{result}$$

Illustrative Example 2. $4.5 \times 4.5 = ?$

SOLUTION:

$$\begin{array}{r} 4.5 \\ 4.5 \\ \hline 20.25 \end{array}$$

169. Division of Numbers Containing Fractions.—

Illustrative Example 1. To divide a mixed number by a whole number
 $64\frac{2}{3} \div 9 = ?$

SOLUTION:

$$\begin{aligned} 64\frac{2}{3} &= \frac{194}{3} \\ 9 &= \frac{27}{3} \\ 194 \div 27 &= 7\frac{5}{27} \end{aligned}$$

or

$$\begin{array}{r} 9 \overline{) 64\frac{2}{3}} \\ 7\frac{5}{27} \end{array}$$

EXPLANATION:

$$64 \div 9 = 7, \text{ and } 1 \text{ over}$$

$$1 + \frac{2}{3} = \frac{5}{3}$$

$$\frac{5}{3} \div 9 = \frac{5}{27}$$

Illustrative Example 2. To divide a whole number by a mixed number.
 $35 \div 7\frac{1}{2} = ?$

SOLUTION:

$$35 = 70 \text{ halves}$$

$$7\frac{1}{2} = 15 \text{ "}$$

$$70 \div 15 = 4\frac{2}{3} \text{ result}$$

CHAPTER XIII

AVERAGES, SIMPLE AND WEIGHTED

170. Kinds of Averages.—In commercial work as well as in other fields, there often occurs a need for knowledge of how to find the average. Unless one has a thorough understanding of this subject he is very liable to arrive at an entirely erroneous conclusion owing to his inability to comprehend the meaning of weighted averages as well as simple averages. For example, it is an easy matter to find the simple average weight of four people, whose weights are 140, 160, 180, and 200 lb., by adding the weights and dividing by four; but were we to find the average wages of four men who earn \$3 each a day, 6 men who earn \$4 each a day, and 5 men who earn \$5 each a day, we would then have to compute the average in a slightly different manner in order to arrive at the correct result.

It is the purpose of this chapter to acquaint the student with the various kinds of averages as well as the proper methods to use in order to arrive at correct conclusions.

171. Simple Average.—This is perhaps best understood by the use of some examples and their solutions. If one will read these carefully and pay particular attention to the solution as indicated he should be able to find the simple average, if the problem is such as to come under this method.

Illustrative Example 1. What is the average weight of a dozen eggs weighing 666 grams?

SOLUTION:

$$666 \text{ grams} \div 12 = 55.5 \text{ grams}$$

Illustrative Example 2. What is the average wage of the following men if 10 men receive \$4 per day; 12 men receive \$3.75 per day; 8 men receive \$4.50 per day; and 5 men receive \$5.25 per day?

EXPLANATION: It is first necessary to find the total wages earned by each group of men, then divide the total of those totals by the total number of men.

SOLUTION:

NUMBER OF MEN	DAY WAGE	TOTAL
10	\$4.00	\$40.00
12	3.75	45.00
8	4.50	36.00
5	5.25	26.25
35		\$147.25

$$\text{Average wage} = \$147.25 \div 35 = \$4.207$$

WRITTEN EXERCISES

1. If six boys weigh respectively 118, 104, 168, 156, 132, and 112 lb., what is their average weight?

2. If a merchant mixes 2 lb. of coffee worth 37¢ a pound, 3 lb. worth 39¢ a pound, and 1 lb. worth 42¢ a pound, what is a pound of the mixture worth?

3. A pay-roll shows 12 hands are employed at \$2.75 per day, 14 hands at \$2.50 per day, 18 hands at \$2.60 per day, and 6 hands at \$5.75 per day. Find the average daily wages.

4. The following clerks in a store made these sales in one day:

CLERK NO.	SALES
122	\$256.75
123	137.80
124	243.60
125	260.80

Find the average sales of these clerks for that day. Which sold above the average? Which below the average?

5. If the expense of running the city of New York is \$316,000,000 in 1920, and the population of the city is approximately 6,000,000, what is the per capita expense?

6. In a certain school of 3,100 pupils, 350 are 13 yr. of age; 800, 14 yr. of age; 750, 15 yr. of age; 600, 16 yr. of age; 550, 17 yr. of age, 40, 18 yr. of age; and 10, 19 yr. of age. Find the average age of the school.

7. Find the difference in cost for a trip from New York to Orange, N. J., if a 10-trip ticket cost \$2.48 and a 50-trip ticket costs \$9.90.

172. Weighted Averages.—These take their name from the fact that they must be weighted or reduced to a common basis in order to obtain a correct average, and unless this is done an entirely erroneous result will be found. This plan is well illustrated in the following example.

Illustrative Example. John Smith has given to William Jones notes as follows: \$150 due May 14; \$200 due June 29; \$500 due July 20. He wishes to pay them all at one time. When shall they be considered due?

EXPLANATION: In order to arrive at a correct solution it is necessary to reduce each of these to a 1-da. basis, for if all the notes were paid May 14 Smith would lose the use of \$200 for 46 da., and of \$500 for 67 da.; or reduced to a 1-da. basis we would have:

$$\begin{array}{rcl} \$200 \times 46 & = & \$ 9,200 \text{ for 1 da.} \\ \$500 \times 67 & = & 33,500 \text{ " 1 " } \\ \hline \text{Total} & = & \$42,700 \text{ " 1 " } \end{array}$$

Smith, therefore, would lose what is equivalent to \$42,700 for 1 da. and is entitled to keep the \$150 + \$200 + \$500 = \$850 as many days after May 14 as are required for the use of \$850 to equal the use of \$42,700 for 1 da., or $\frac{42,700}{850} = 50.2$ da. Hence the equated time for paying all the notes is 50 da. after May 14 or July 3, or arranged as follows:

$$\begin{array}{rcl} \$150 \times 0 & = & \$ 0,000 \\ 200 \times 46 & = & 9,200 \\ 500 \times 67 & = & 33,500 \\ \hline \$850 &) & \$42,700 \\ & & 50.2 \end{array}$$

WRITTEN EXERCISES

1. Find the equated time for the payment of \$250 due in 3 mo., \$400 due in 6 mo., and \$700 due in 8 mo.

2. Find the equated time for the payment of \$300 due in 30 da., \$500 due in 60 da., and \$200 due in 90 da.

3. Find the equated time for the payment of \$275 due June 21, \$175 due July 16, \$200 due Aug. 6, and \$150 due Sept. 3.

4. A owes B \$200 due in 10 mo. If he pays \$120 in 4 mo., when should he pay the balance?

SOLUTION: By paying \$120 in 4 mo. A loses the use of \$120 for 6 mo., which is equal to the use of \$720 for 1 mo. Therefore, he is entitled to keep the balance, \$80, $\frac{6}{9}$ mo., or (reducing the fraction) 9 mo. after its maturity.

5. A owes B \$2,000 payable in 4 mo., but at the end of 1 mo. he pays him \$500, at the end of 2 mo. \$500, and at the end of 3 mo. \$500. In how many months is the balance due?

6. A man bought Feb. 11, a bill of goods amounting to \$1,700, on 4 mo. credit; but he paid Mar. 22, \$400; Apr. 20, \$220; and May 10, \$300. When is the balance due?

7. Find the equated time of maturity of each of the following bills, and the amount due at settlement including interest at 6%.

JOHN DOE TO WILLIAM PRICE, JR.

Apr. 5	To mdse. on 4 mo. credit.....	\$120.50
Apr. 15	To " " 3 " "	87.33
May 7	To " " 3 " "	218.17
May 21	To " " 4 " "	317.00

\$743.00

Paid Oct. 18

HINT: Find the equated time for payment, reckoning from July 15, which is the earliest date that any item becomes due, and find the interest on the whole bill from the equated time to Oct. 18.

8. If two men begin business together with \$6,000 borrowed capital and at the end of 6 yr. their net worth is \$18,600, what was their average annual gain?

9. If Mr. Williams deposits on Apr. 1, in a bank that pays interest on average monthly balances, the sum of \$2,000, and on Apr. 21 he reduces this sum to \$1,600, determine his average balance for April.

SOLUTION:

$$\frac{20 \times \$2,000 + 10 \times \$1,600}{30} = \frac{\$56,000}{30} = \$1,866.67$$

10. Find the average daily balance at the end of the month of a man's bank account from the following information:

DEPOSITS	CHECKED OUT
\$300 Jan. 1	\$50 Jan. 5
	50 " 10
	50 " 15
	50 " 20
	50 " 25

11. On the organization of a partnership, A invests \$12,000, but withdraws \$2,000 at the end of 8 mo.; B invests \$6,000, withdrawing \$3,000 after 6 mo. How should the 1st year's gains of \$4,750 be apportioned?

SOLUTION:

$$\begin{array}{lcl} \text{A's investment of } \$12,000 \text{ for 8 mo.} & = & \$ 96,000 \text{ for 1 mo.} \\ \text{A's " " 10,000 " 4 " } & = & 40,000 \text{ " 1 " } \end{array}$$

$$\text{A's average investment} = \$136,000 \text{ " 1 mo.}$$

$$\begin{array}{lcl} \text{B's investment of } \$ 6,000 \text{ for 6 mo.} & = & \$ 36,000 \text{ for 1 mo.} \\ \text{B's " " 3,000 " 6 " } & = & 18,000 \text{ " 1 " } \end{array}$$

$$\text{B's average investment} = \$ 54,000 \text{ " 1 mo.}$$

$$\text{A's average} + \text{B's average} = \$190,000 \text{ " 1 "}$$

$$\text{A's share} = \frac{136}{190} = \frac{68}{95}$$

$$\text{B's " } = \frac{54}{95}$$

$$\frac{1}{95} \text{ of profit of } \$4,750 = \$50$$

$$\frac{68}{95} = 68 \times \$50 = \$3,400 = \text{A's profit}$$

$$\frac{27}{95} = 27 \times \$50 = \$1,350 = \text{B's "}$$

Using 6%:

$$\text{Interest on } \$12,000 \text{ for 8 mo.} = \$480$$

$$\text{" " 10,000 " 4 " } = 200$$

$$\text{A's invested earning power} = \$680$$

$$\text{Interest on } \$6,000 \text{ for 6 mo.} = \$180$$

$$\text{" " 3,000 " 6 " } = 90$$

$$\text{B's invested earning power} = \$270$$

Total earning power, $\$680 + \$270 = \$950$, of which A's investment represents $\frac{6}{9}$ and B's $\frac{2}{9}$.

12. X and Y form a partnership. X invests \$12,000 for 9 mo., and then adds \$4,000. Y invests \$28,000, but withdraws \$8,000 at the end of 4 mo. At the end of a year their accounts stand as follows:

GOODS OF DEPT. A

Cost.....	\$12,480.00	Sales.....	\$10,537.00
		On hand.....	7,300.00

GOODS OF DEPT. B

Cost.....	\$ 4,264.00	Sales.....	\$ 7,172.80
		On hand.....	2,250.00

GOODS OF DEPT. C

Cost.....	\$11,384.00	Sales.....	\$14,436.40
		On hand.....	1,930.00

GENERAL EXPENSE

Cost.....	\$ 2,592.36
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Apportion the profits according to the average investment.

13. The following is a memorandum of flour stored by B. G. Jackson with the Heights Storage Co. at 4¢ per bbl. per term of 30 da., average storage. What was the amount of the bill?

SOLUTION:

DATE	RECEIPTS	DELIVERIES	BALANCE	TIME IN STORAGE	IN STORAGE FOR 1 DA.
Feb. 6.....	200 bbl.		200 bbl.	6 da.	1,200 bbl.
" 12.....		100 bbl.	100 "	9 "	900 "
" 21.....	150 "		250 "	15 "	3,750 "
Mar. 8.....	400 "		650 "	1 "	650 "
" 9.....		150 "	500 "	12 "	6,000 "
" 21.....		300 "	200 "	8 "	1,600 "
" 29.....	200 "		400 "	6 "	2,400 "
Apr. 4.....		400 "	0 "		16,500 bbl.

The storage items are equivalent to the storage of 1 bbl. for 16,500 da.
 $16,500 \text{ da.} = 550 \text{ terms of } 30 \text{ da. each.}$ At 4¢ per term, the storage =
 $550 \times 4¢ = \$22,$

14. The Dobson Storage Co. received and delivered on account of W. T. Johnson sundry barrels of flour as follows: received Nov. 10, 2,000 bbl., Nov. 20, 1,200 bbl., Dec. 15, 800 bbl., Jan. 20, 2,000 bbl.; delivered Dec. 2, 1,200 bbl., Dec. 28, 1,400 bbl., Jan. 24, 500 bbl., Feb. 4, 800 bbl., Mar. 30, 300 bbl. If the charges were $4\frac{1}{2}\text{¢}$ per bbl. per term of 30 da. average storage, what was the amount of the bill?

CHAPTER XIV

THE PROGRESSIONS

173. Arithmetic Progression.—This name is given to a certain series of numbers, each term of which is formed by adding a constant quantity, called the difference, to the preceding term; for example, 1, 4, 7, 10, etc. Therefore, to find the difference, subtract any term from the following term. It is useful at times in order to find the total of a given series of numbers or to find any particular term of that series. For example, if a car, going down an inclined plane, travels in successive seconds, 2 ft., 6 ft., 10 ft., 14 ft., etc., how far will it go in 30 seconds? The long method would be to set down all the 30 numbers and add them; but we shall see that it can be done in a much quicker and easier way.

174. Quantities and Symbols.—There are certain well-established symbols which are used in the consideration of an arithmetic series of numbers, which are:

a = the first term
 d = the common difference
 l = the last term
 n = the number of terms
 s = the sum of the terms

175. General Form of an Arithmetic Progression.—This general form is $a, a + d, a + 2d, a + 3d, \dots$. Therefore, coefficient of d in each term is 1 less than the number of the term.

Illustrative Example 1.

$$\begin{aligned}
 7\text{th term} &= a + 6d \\
 12\text{th} \quad " &= a + 11d \\
 n\text{th} \quad " &= a + (n - 1)d \\
 \text{Hence } l &= a + (n - 1)d \quad \text{I}
 \end{aligned}$$

Also:

$$\begin{array}{ll}
 s = a + (a + d) + (a + 2d) + \dots + (l - d) + l & (1) \text{ or writing this in} \\
 s = l + (l - d) + (l - 2d) + \dots + (a + d) + a & (2) \text{ the reverse order} \\
 \hline
 2s = (a + l) + (a + l) + (a + l) + \dots + (a + l) + (a + l) & \text{By adding (1) and} \\
 \text{or } 2s = n(a + l) & (2). \\
 s = n/2(a + l) & \text{II} \\
 s = n/2[2a + (n - 1)d] & \text{III By substituting I in II}
 \end{array}$$

Illustrative Example 2. Find the 12th term and the sum of this series of numbers 5, 3, 1, $-1 \dots$

$$\begin{array}{lll}
 \text{SOLUTION:} & l = a + (n - 1)d & a = 5 \\
 & = 5 + (12 - 1)(-2) & d = -2 \\
 & = 5 + (-22) & n = 12 \\
 & = -17 & \\
 \text{Sum} & = \frac{n}{2}(a + l) & a = 5 \\
 & = \frac{1}{2}(5 - 17) & d = -2 \\
 & = 6(-12) & n = 12 \\
 & = -72 &
 \end{array}$$

WRITTEN EXERCISES

- Find the 8th term of the series 3, 7, 11, $\dots$
- Find the sum of the first 30 odd numbers.
- If a man saves \$100 in his 20th yr., \$150 the next year, \$200 the next, and so on through his 50th yr., how much will he save in all?
- If a body falls 16.1 ft. in 1 second, three times as far in the next second, and 80.5 ft. in the 3d second, and so on, how far will it fall in 6 seconds? In 20 seconds?
- If a clerk received \$900 for his 1st year's salary, and a regular yearly increase of \$50 for the next 10 yr., find his salary for the 11th yr. Find the total salary for the 11 yr.
- Find the sum of 1¢, 2¢, 3¢, etc., to \$1 inclusive.
- Find the sum of the first 30 even numbers. Of the first 100 numbers.
- Find the sum of $\frac{1}{2}$ ¢, 1¢, $1\frac{1}{2}$ ¢, etc., to \$1 inclusive.

9. Compare your answer in Exercise 8 with that in Exercise 6, and find the gain if lottery tickets are sold as by Exercise 8, rather than by Exercise 6.

10. A man invests his savings in the shares of a building and loan association, depositing \$1,000 the 1st yr. At the beginning of the 2d yr. he is credited with \$60 interest on the amount deposited the 1st yr., and pays in only \$940, making his total credit \$2,600. At the beginning of the 3d yr. he is credited with \$120 interest, and pays in \$880 cash, etc. What is his credit at the end of 10 yr., and how much cash has he paid in?

176. Geometrical Progression.—This is a series of numbers, each term of which is formed by multiplying the preceding term by a constant called the **ratio**.

The series 1, 3, 9, 27, etc., is one in which the ratio is 3. The same symbols except d are used as in the arithmetic progression with the addition of r for the ratio.

The type series is $a, ar, ar^2, ar^3, ar^4, \dots$. Hence the exponent of r in each term is 1 less than the number of the term.

Illustrative Example 1.

$$\begin{aligned} 10\text{th term} &= ar^9 \\ 15\text{th } " &= ar^{14} \\ n\text{th } " &= ar^{n-1} \\ \therefore l &= ar^{n-1} \end{aligned}$$

I

In deriving an equation for the sum, we know, also:

$$\begin{aligned} s &= a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} & (1) \text{ Now multiply (1) by } r \\ rs &= ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} + ar^n & (2) \end{aligned}$$

$$\begin{aligned} rs - s &= ar^n - a & \text{Subtracting (1) from (2)} \\ s(r - 1) &= ar^n - a \end{aligned}$$

$$s = \frac{ar^n - a}{r - 1} \quad \text{II}$$

Multiplying $l = ar^{n-1}$ by r , we get $rl = ar^n$

Substituting rl for ar^n in (II).

$$s = \frac{rl - a}{r - 1} \quad \text{III}$$

Illustrative Example 2. Find the 8th term and the sum of the 8 terms of the geometric progression, 1, 3, 9, 27.....

$$\begin{aligned}
 \text{SOLUTION: } l &= ar^{n-1} & a &= 1 \\
 &= 1(3)^7 & r &= 3 \\
 &= 2,187 & n &= 8 \\
 s &= \frac{rl - a}{r - 1} \\
 &= \frac{3 \times 2,187 - 1}{3 - 1} \\
 &= \frac{6,561 - 1}{2} \\
 &= 3,280
 \end{aligned}$$

Illustrative Example 3. Find the 10th term and the sum of 10 terms of the geometrical progression, 4, -2, 1, $-\frac{1}{2}$

$$\begin{aligned}
 \text{SOLUTION: } l &= ar^{n-1} & a &= 4 \\
 &= 4(-\frac{1}{2})^9 & r &= -\frac{1}{2} \\
 &= 4(-\frac{1}{512}) \\
 &= -\frac{1}{128} \\
 s &= \frac{(-\frac{1}{2})(-\frac{1}{128}) - 4}{-\frac{1}{2} - 1} \\
 &= \frac{3\frac{1}{128}}{1\frac{1}{2}}
 \end{aligned}$$

WRITTEN EXERCISES

1. Find r , then find the 6th term in the series 2, 6, 18.....
2. Find the 9th term in the series $2, 2\sqrt{2}, 4, \dots$
3. A ship was built at a cost of \$70,000. Her owners at the end of each year deducted 10% from her value as estimated at the beginning of the year. What is her estimated value at the end of 10 yr.?
4. The population of a city increases in 4 yr. from 10,000 to 14,641. What is the rate of increase if

$$r = \sqrt[n-1]{\frac{l}{a}}$$

5. The population of the United States in the year 1900 was 76,300,000. If this should increase 50% every 25 yr., what would the population be in the year 2000?
6. If the annual depreciation of a building is estimated at 4% of its value at the beginning of each year and the building cost \$25,000, what is its estimated value at the end of 20 yr.?

7. A machine costing \$9,000 depreciates 7% of its value at the beginning of each year. Find its estimated value at the end of 8 yr.

8. In making an inventory at the close of each year, a manufacturer deducted 10% from the value of his machinery at the previous inventory, because of deterioration. The machine cost \$20,000. What was the value at the end of the 5th yr.?

HINT: Value at end of 1st yr. = $.90 \times \$20,000$
 " " " " 2d " = $.90^2 \times \$20,000$

What was the common ratio?

9. A boy puts \$100 in a savings bank, which pays 3% compound interest, compounded annually. What does it amount to at the end of 6 yr.?

HINT: Value end of 1st yr. = $1.03 \times \$100$

10. The population of a city is 100,000, and increases 10% each year for 10 yr. Find the population in 10 yr.

11. A person has two parents, each of his parents has two parents, and so on. How many ancestors has a person, going back ten generations, counting his grand parents as the first generation and assuming that each ancestor is an ancestor in only one line of descent?

CHAPTER XV

LOGARITHMS

177. Logarithms.—Many kinds of commercial work, as well as academic or technical work, require computation by the use of logarithm tables. Such forms are multiplication, division, raising numbers to required powers, and extracting roots of numbers. This work is made much easier as well as much quicker at times by the use of these tables or calculating devices. The tables are not difficult to understand if one will study thoroughly the explanation of their use.

178. Calculations Made Through the Use of Exponents.—Whenever numbers which are the powers of one number, as for instance, of 2, are to be multiplied, divided, raised to powers, or have their roots extracted, these operations can be performed very quickly if a table containing the various powers of the numbers has been prepared beforehand.

TABLE		
POWER OF 2		NUMBER
2^1	=	2
2^2	=	4
2^3	=	8
2^4	=	16
2^5	=	32
2^6	=	64
2^7	=	128
2^8	=	256
2^9	=	512
2^{10}	=	1024

TABLE

POWER OF 2		NUMBER
2^{11}	=	2048
2^{12}	=	4096
2^{13}	=	8192
2^{14}	=	16384
2^{15}	=	32768
2^{16}	=	65536
2^{17}	=	131072
2^{18}	=	262144
2^{19}	=	524288
2^{20}	=	1048576

WRITTEN EXERCISES

1. Calculate 16×64 by the use of the table.

SOLUTION: $16 = 2^4$
 $64 = 2^6$
 $16 \times 64 = 2^4 \times 2^6$
 $= 2^{10}$ Same as $x^4 x^6 = x^{10}$
 $= 1,024$ (From the table)

2. Calculate $8 \times 128 \times 16$ by the use of the table.

3. Calculate $16,384 \div 256$ by the use of the table.

SOLUTION: $16,384 = 2^{14}$
 $256 = 2^8$
 $16,384 \div 256 = 2^{14} \div 2^8$
 $= 2^6$ Same as $x^{14} \div x^8 = x^6$
 $= 64$ (From the table)

4. Apply the table to:

- (a) $1,024 \div 16$
 (b) $512 \div 64$
 (c) $32,768 \div 1,024$

5. Square 32 by the use of the table.

SOLUTION: $32^2 = (2^5)^2$
 $= 2^{10}$ Same as $(x^5)^2 = x^{10}$
 $= 1,024$ (From the table)

6. Apply the table to the following:

(a) 32^3

(b) 64^2

(c) 32^4

7. Find the square root of 256 by the use of the table.

SOLUTION: $\sqrt[2]{256} = \sqrt[2]{2^8}$
 $= 2^4$ Same as $\sqrt{x^8} = x^4$
 $= 16$

EXPLANATION: Divide the root (2) into the power (8) and it gives 4

8. Apply the table to find the square root of 16,384.

9. " " " " " " cube " " 32,768.

10. " " " " " " fourth " " 4,096.

11. " " " " " " fifth " " 1,024.

12. Solve $\frac{64 \times 256 \times 16}{32 \times 512}$ by the table.

13. Calculate $1,024 \times 16$ " " "

14. Calculate 512×64 " " "

179. Logarithms Are Exponents.—The **logarithm** of a given number is the exponent of the power to which a base number must be raised to produce this number.

In logarithms three different numbers are always involved:

1. A number.
2. Its logarithm.
3. The base used.

Illustrative Example. In $3^2 = 9$, we can say 2 is the logarithm of 9 to the base 3. Similarly, since $3^4 = 81$, we can say 4 is the logarithm of 81 to the base 3. Therefore, $B^L = N$, we can say L is the logarithm of N to the base B .

180. A System of Logarithms.—This is a set of numbers with their logarithms all taken to the same base. Notice that the logarithm of 1 in any system is 0, since $a^0 = 1$.

SYSTEM OF LOGARITHMS WITH BASE 2

NUMBER	LOGARITHM	REASON	NUMBER	LOGARITHM	REASON
1	0	$2^0 = 1$			
2	1	$2^1 = 2$	$\frac{1}{2}$	- 1	$2^{-1} = \frac{1}{2}$
3	1.585	$2^{1.585} = 3$			
4	2	$2^2 = 4$	$\frac{1}{4}$	- 2	$2^{-2} = 2^{\frac{1}{2}} = \frac{1}{4}$
5	2.3223	$2^{2.3223} = 5$			
8	3	$2^3 = 8$	$\frac{1}{8}$	- 3	$2^{-3} = \frac{1}{(2)^3} = \frac{1}{8}$

The student need not know exactly how decimal logarithms like 1.585 are found. Originally they were found by a long process of extracting roots. Since logarithms are exponents, they may be interpreted as such. Thus in the equation $2^{1.585} = 3$, we see that the 1.585th power of the 10,000th root of 2 equals 3, and if these operations were actually performed on 2, the result would be 3.

181. Notation and Terms. To avoid writing long exponents, such an equation as $2^{1.5850} = 3$ is changed into $\log_2 3 = 1.5850$, and is read "logarithm of 3 to the base 2 equals 1.5850." The subscript indicating the base is usually omitted when 10 is the base.

The integral part of the logarithm is called its **characteristic** and the decimal part the **mantissa**.

WRITTEN EXERCISES

Express the following in the language of logarithms:

1. $2^4 = 16$

SOLUTION: $\log_2 16 = 4$. Read, logarithm of 16 to base 2 is 4.

2. $3^3 = 27$

6. $3^{-2} = \frac{1}{9}$

3. $10^3 = 1,000$

7. $10^2 = 100$

4. $4^2 = 16$

8. $10^{-1} = \frac{1}{10}$

5. $2^5 = 32$

9. $10^{-4} = .0001$

Express in the language of exponents, and find the value of each:

- | | |
|-------------------------------|----------------------------------|
| 10. $\log_2 8 = 3$ | 17. $\log_3 27 = x$ |
| 11. $\log_4 64 = 3$ | 18. $\log_4 64 = x$ |
| 12. $\log_5 25 = 2$ | 19. $\log_4 \frac{1}{16} = x$ |
| 13. $\log_8 4 = \frac{2}{3}$ | 20. $\log_3 \frac{1}{9} = x$ |
| 14. $\log_{10} .01 = -2$ | 21. $\log_3 \frac{1}{81} = x$ |
| 15. $\log_9 27 = \frac{3}{2}$ | 22. $\log_{10} \frac{1}{10} = x$ |
| 16. $\log_3 9 = x$ | 23. $\log_{10} .01 = x$ |
| Find the value of x | 24. $\log_{10} .001 = x$ |
| SOLUTION: $3^x = 9$ | 25. $\log_2 32 = x$ |
| $3^x = 3^2$ | 26. $\log_2 \frac{1}{32} = x$ |
| $\therefore x = 2$ | 27. $\log_4 8 = x$ |
| | 28. $\log_8 16 = x$ |

29. What is the logarithm of 9 in a system whose base is 3? Of 81? Of 27? Of 3? Of $\frac{1}{9}$?

30. What is the logarithm of 256 in a system whose base is 16? Of 16? Of 4? Of 8? Of 64?

31. What is the logarithm of 100 in a system whose base is 10? Of 1,000? Of 100,000? Of $\frac{1}{10}$? Of .01? Of 1? Of .001?

32. What is the logarithm of 81 in a system whose base is 27? Of 3? Of 9? Of 243? Of $\frac{1}{27}$? Of $\frac{1}{9}$? Of $\frac{1}{81}$?

182. Briggsian or Common System of Logarithms.—This system uses 10 for the base.

Illustrative Example.

Since 10^4	=	10,000	then	$\log 10,000$	=	4
10^3	=	1,000	“	$\log 1,000$	=	3
10^2	=	100	“	$\log 100$	=	2
10^1	=	10	“	$\log 10$	=	1
10^0	=	1	“	$\log 1$	=	0
10^{-1}	=	.1	“	$\log .1$	=	-1
10^{-2}	=	.01	“	$\log .01$	=	-2
10^{-3}	=	.001	“	$\log .001$	=	-3

183. Positive Characteristics. You will observe that any number between 1,000 and 10,000 has for its logarithm, 3 + a decimal. Any number between 100 and 1,000 has

2 + a decimal; and between 10 and 100, 1 + a decimal; and a number between 1 and 10 has 0 + a decimal. Hence, in general, the characteristic of the logarithm of any number greater than 1, in the Briggsian or common system of logarithms, is 1 less than the number of places at the left of the decimal point. Thus the characteristic of 729.4 is 2; of 7,460 is 3; of 3.96 is 0.

184. No Change in Mantissa When Decimal Point is Moved.—In the common system, in which the base is 10, the mantissas do not change when the decimal point is moved. To understand why this is true, we take $10^{1.038} = 1.27$, and multiply or divide both members of this equation by $10^2 = 100$, or by $10^1 = 10$. Recalling that when x^2 is multiplied by x^5 we obtain x^{2+5} or x^7 , and when x^8 is divided by x^3 we obtain x^{8-3} or x^5 , then by the same process of reasoning we have:

$$\begin{array}{lll} 10^{2.1038} & = & 127 \text{ or } \log 127 = 2.1038 \\ 10^{1.1038} & = & 12.7 \text{ " } \log 12.7 = 1.1038 \\ 10^{1.038-1} & = & .127 \text{ " } \log .127 = \bar{1}.1038 \text{ or } 9.1038-10 \\ 10^{1.038-2} & = & .0127 \text{ " } \log .0127 = \bar{2}.1038 \text{ or } 8.1038-10 \end{array}$$

The minus sign **over** the characteristic at the right belongs to the characteristic only. Thus, by regarding the characteristic only as changing in signs, mantissas stay the same no matter where the decimal point in the number is changed to, and **mantissas are always positive**.

185. Negative Characteristics.—Any number between .001 and .01, having two ciphers (or zeros) before the first significant (i.e., first figure other than 0) figure, has $\bar{3}$ for its characteristic, since its logarithm lies between $\bar{3}$ and $\bar{2}$ and

the mantissa added is positive. Any number between .01 and .1 has $\bar{2}$ for its characteristic, since its logarithm lies between $\bar{2}$ and $\bar{1}$, and the mantissa added is positive; also, any number between .1 and 1, there being no cipher before its first significant figure, has $\bar{1}$ for its characteristic, since its logarithm lies between $\bar{1}$ and 0 and the mantissa added is positive. Hence, in general, the characteristic of any number less than 1 is **one more** than the number of ciphers between the decimal point and the first significant figure, and is **negative**. Thus, the characteristic of the logarithm of .00468 is $\bar{3}$; of .7396 is $\bar{1}$; of .000076 is $\bar{5}$.

186. Explanation of a Logarithm Table.—In the logarithm table the left-hand column is a column of ordinary numbers. The first two figures of the given number whose mantissa is sought are found in this column. In the top row are the figures from 0 to 9. The third number is found there.

Hence, to obtain the mantissa of 364, we take 36 in the first column and look along the row beginning with 36 until we come to the column headed 4. The mantissa thus obtained is 5611. To find the mantissa of 2,710 we find the mantissa of 271, and the mantissa of 7 is the same as that of 70 or 700.

187. To Find the Mantissa of a Number Containing More than Three Figures (Interpolation).—Find the mantissa for the first three figures and add a correction for the remaining figures. This correction is computed on the assumption that the differences in logarithms are proportional to the differences in the numbers to which they belong. Though this proportion is not strictly accurate, it is sufficiently accurate for practical purposes.

Illustrative Example: Find the mantissa for 1,581.47.

$$\begin{array}{r} \text{mantissa for } 159 = .2014 \\ \text{" " } 158 = .1987 \\ \hline \text{difference for } 1 = .0027 \end{array}$$

$$\begin{array}{r} \text{mantissa for } 158 = .1987 \\ .0027 \times .147 = .0004 \\ \hline \text{mantissa for } 1,581.47 = .1991 \end{array}$$

The difference between the mantissas of two successive numbers is called the **tabular difference**. Hence, to find from the table the mantissa for a number containing more than three figures: Obtain from the table the mantissa for the first three figures, and also that for the next higher number, and subtract. **Multiply the difference between the two mantissas by the remaining figures with a decimal point at their left, and add the result to the mantissa for the first three figures.**

188. To Find the Logarithm of a Given Number.—Determine the characteristic. Neglect the decimal point (in the given number) and obtain from the table the mantissa for the given figure.

Illustrative Example 1. Find the logarithm of 3.6257.

SOLUTION: The characteristic of 3.6257 is 0, since 1 (the number of places at the left of the decimal point) $- 1 = 0$.

$$\begin{array}{r} \text{mantissa of } 363 = .5599 \\ \text{" " } 362 = .5587 \\ \hline \text{difference for } 1 = .0012 \end{array}$$

$$\begin{array}{r} \log \text{ of } 3.62 = 0.5587 \\ .0012 \times .57 = .0007 \\ \hline \log \text{ of } 3.6257 = 0.5594 \end{array}$$

Illustrative Example 2. Find the logarithm of .078546.

SOLUTION: The characteristic of .078546 is 2, since 1 (the number of ciphers at the right of the decimal point) $+ 1 = 2$.

$$\begin{array}{r} \text{mantissa of } 786 = .8954 \\ \text{" " } 785 = .8949 \\ \hline \text{difference for } 1 = .0005 \end{array}$$

$$\begin{array}{r} \log \text{ of } .0785 = \bar{2}.8949 \\ = 8.8949 - 10 \\ .0005 \times .46 = .0002 \\ \hline \log \text{ of } .078546 = 8.8951 - 10 \end{array}$$

EXPLANATION: Instead of using the 2.8949, we change the -2 to $8 - 10$, which it equals.

WRITTEN EXERCISES

Find the logarithms of the following numbers whose mantissas are found directly in the table. Common fractions and mixed numbers must first be reduced to decimals.

- | | | |
|---------------------|---------------------|--------------|
| 1. 400 | 10. $47\frac{1}{5}$ | 18. 70,000 |
| 2. 4 | 11. 699 | 19. 25.4 |
| 3. .0372 | 12. $\frac{1}{25}$ | 20. 3.56 |
| 4. $\frac{1}{4}$ | 13. $4\frac{1}{2}$ | 21. 356 |
| 5. 37 | 14. $\frac{7}{8}$ | 22. 3,560 |
| 6. 40 | 15. $12\frac{1}{2}$ | 23. 5,670 |
| 7. 7,000 | 16. 3,680 | 24. .00046 |
| 8. .000029 | 17. .000451 | 25. .0000564 |
| 9. $.06\frac{1}{2}$ | | |

189. How to Use Tables of Proportionate Parts.—These tables are time-savers in finding the mantissas of given numbers. They are used as follows:

Illustrative Example. If the number is 36.54 and we desire its logarithm, we note that its characteristic is 1.

Then we find the difference between the mantissa of 365 and that of 366, which is 12. Looking down the **extra-digit** column headed 12 until we find 4 (the fourth figure in the number), and following across to the right, we find 4.8. We add this difference to the last figure of the mantissa of 365, which gives 5,628 (as 4.8 gives 5 with the .8 correction made to the next figure at the left).

$$\log 36.54 = 1.5628$$

WRITTEN EXERCISES

To find the logarithms of numbers whose mantissas are not in the table.

Find the logarithm of the following:

- | | | |
|-----------------------|--------------|-------------|
| 1. 921.5 | 6. .6757 | 11. 31.393 |
| 2. 3.1416 | 7. .09496 | 12. 48387. |
| 3. $\sqrt{2} = 1.414$ | 8. 4.288 | 13. 7.3165 |
| 4. $\sqrt{3}$ | 9. .00005023 | 14. .019698 |
| 5. 1079 | 10. .0002625 | 15. 810.39 |

190. How to Find Antilogarithms.—Since the characteristic depends only on the position of the decimal point and not on the figures forming the given number, the characteristic is neglected at the outset of the process of finding the antilogarithm.

1. If the given mantissa can be found in the table: Take from the table the figures corresponding to the mantissa of the given logarithm; use the characteristic of the given logarithm to fix the decimal point in the number obtained from the table.

Illustrative Example. Find the number whose logarithm is 1.4425.

SOLUTION: The figures corresponding to the mantissa .4425 are 277. Since the characteristic is 1, there are two figures at the left of the decimal point.

Therefore, if

$$\begin{aligned}\log x &= 1.4425 \\ x &= 27.7\end{aligned}$$

2. If the given mantissa does not occur in the table: Obtain from the table the next lower mantissa with the corresponding three figures of the antilogarithm. Subtract the tabular mantissa from the given mantissa; divide the latter difference by the difference between the next lower and the next higher mantissa in the table; **annex** this quotient to the three figures of the antilogarithm already obtained from the table. Use the characteristic to place the decimal point in the result.

Illustrative Example 1. Find the number (or antilogarithm) whose logarithm is 2.4237.

SOLUTION: .4237 is not in the table; the next lower is .4232. The difference between them is .0005. If a difference of 17 in the last two figures of the mantissa makes a difference of 1 in the third figure of the number (antilog), a difference of 5 in the last figure of the mantissa will make a difference of $\frac{5}{17}$ of 1, or .294, with respect to the third figure of the number.

$$\begin{aligned}\text{Hence if} \quad \log x &= 2.4237 \\ x &= 265.294\end{aligned}$$

Illustrative Example 2. If $\log x = 7.2661 - 10$, find x .

SOLUTION: The nearest less mantissa is .2660, of which the number is 1845.

$$\begin{aligned}\text{The tabular difference} &= 2 \\ 1 \div 2 &= .5 \\ \therefore x &= .0018455\end{aligned}$$

191. To Find the Number by Use of Proportionate Parts.

—If the logarithm is given the number may be found by use of proportionate parts. The method is outlined in the following:

Illustrative Example. Find the number of which the logarithm is 1.8678.

SOLUTION: Find the mantissa just below this, and put down the corresponding first three figures of the number, or 737.

Find the difference between the mantissa just below the given mantissa and just above the given mantissa, which in this case is 6.

Find the difference between the given mantissa and the next lower mantissa, which in this case is 3.

Find the proportionate parts column headed 6, follow down it until you come to the figure 3 (or the figure nearest to it in value), then follow across to the left-hand column headed extra digit, which in this case is 5.

Annex this figure to the first three figures (737 in this case), making 7,375.

Use the characteristic to point this off, giving the number whose logarithm is 1.8678 as 73.75.

192. Table of Logarithms.—The following is a table of logarithms of numbers, such as may be applied to the problems in this book. The table of proportionate parts is at the right.

No.	0	1	2	3	4	5	6	7	8	9	Prop. Parts				
											Extra digit.	Difference			
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	14	13		
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254				1.4	2.6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319				2.8	3.9
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382				4.2	5.2
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445				5.6	6.5
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	7.0	7.8			
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	8.4	9.1			
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	9.8	10.4			
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	11.2	11.7			
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	12.6				
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	2	12	11		
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859				1.2	2.2
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915				2.4	3.3
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971				3.6	4.4
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025				4.8	5.5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	6.0	6.6			
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	7.4	7.7			
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	8.6	8.8			
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	9.6	9.9			
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	10.8				
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	3	10	9		
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390				1.0	1.8
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440				2.0	2.7
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489				3.0	3.6
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538				4.0	4.6
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5.0	5.1			
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	6.0	6.3			
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	7.0	7.2			
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	8.0	8.1			
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	9.0				
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	4	8	7		
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863				0.8	0.7
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908				1.6	1.4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952				2.4	2.1
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996				3.2	2.8
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	4.0	3.5			
101	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	5.0	4.2			
102	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	6.0	4.9			
103	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	7.0	5.6			
104	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	8.0	5.6			
105	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	9.0	7.2			
106	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290					
107	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330					
108	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370					
109	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410					

No.	0	1	2	3	4	5	6	7	8	9	Prop. Parts	
											Ex. dig.	Difference
110	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449		
111	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488		
112	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527		
113	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565		
114	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603		
115	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641		
116	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678		
117	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715		
118	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752		
119	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788		
120	0792	0795	0799	0803	0806	0810	0813	0817	0821	0824		
121	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860		
122	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896		
123	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931		
124	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966		
125	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000		
126	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035		
127	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069		
128	1072	1075	1079	1082	1086	1089	1093	1096	1099	1103		
129	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136		
130	1139	1143	1146	1149	1153	1156	1159	1163	1166	1169		
131	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202		
132	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235		
133	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268		
134	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300		
135	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332		
136	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364		
137	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396		
138	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427		
139	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458		
140	1461	1464	1467	1471	1474	1477	1480	1483	1486	1489		
141	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520		
142	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550		
143	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581		
144	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611		
145	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641		
146	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670		
147	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700		
148	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729		
149	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758		
150	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787		
151	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816		
152	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844		
153	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872		
154	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901		

No.	0	1	2	3	4	5	6	7	8	9	Prop. Parts	
155	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	Extra digit	Difference
156	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956		
157	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984		
158	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011		
159	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038		
160	2041	2044	2047	2049	2052	2055	2057	2060	2063	2066		
161	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092		
162	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119		
163	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	1	3
164	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2	0.6
165	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	3	0.9
166	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	4	1.2
167	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	5	1.5
168	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	6	1.8
169	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	7	2.1
170	2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	8	2.4
171	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	9	2.7
172	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378		
173	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	1	0.2
174	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2	0.4
175	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	3	0.6
176	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	4	0.8
177	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	5	1.0
178	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	6	1.2
179	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	7	1.4
180	2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	8	1.6
181	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	9	1.8
182	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622		
183	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	1	0.3
184	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	2	0.6
185	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	3	0.9
186	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	4	1.2
187	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	5	1.5
188	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	6	1.8
189	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	7	2.1
190	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	8	2.4
191	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	9	2.7
192	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853		
193	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	1	0.2
194	2878	2880	2883	2885	2887	2889	2891	2894	2896	2898	2	0.4
195	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	3	0.6
196	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	4	0.8
197	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	5	1.0
198	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	6	1.2
199	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	7	1.4
200	3010	3012	3015	3017	3019	3021	3023	3025	3028	3030	8	1.6
											9	1.8

WRITTEN EXERCISES

Find the numbers (or antilogarithms) which correspond to the following logarithms:

- | | |
|------------------|------------------|
| 1. 0.8189 | 16. 1.81418 |
| 2. 7.60640 - 10 | 17. 1.41863 |
| 3. 1.87670 | 18. 0.98349 |
| 4. 2.67600 | 19. 9.22321 - 10 |
| 5. 3.98260 | 20. 5.00400 |
| 6. 8.79540 - 10 | 21. 2.34578 |
| 7. 6.59930 - 10 | 22. 1.63350 |
| 8. 9.94370 - 10 | 23. 0.57750 |
| 9. 0.77810 | 24. 3.92430 |
| 10. 5.45710 - 10 | 25. 9.79730 - 10 |
| 11. 1.30190 | 26. 7.70070 - 10 |
| 12. 4.25270 - 10 | 27. 1.49000 |
| 13. 2.01590 | 28. 1.89040 |
| 14. 3.72640 - 10 | 29. 2.45270 |
| 15. 4.49290 | 30. 9.64020 - 10 |

193. Computation by the Use of Logarithms.—It is shown in algebra that,

$$x^2x^3 = x^5$$

and that,

$$a^xa^y = a^{x+y}$$

It is shown that,

$$(x^2)^4 = x^8,$$

and that

$$(a^x)^p = a^{px}$$

It can also be shown that:

$$1. \log(mn) = \log m + \log n$$

$$\text{For if } m = 10^x \quad \text{Then } \log m = x$$

$$\text{And if } n = 10^y \quad \text{“} \quad \log n = y$$

$$mn = 10^{x+y} \quad \text{or } \log mn = x + y$$

$$= \log m + \log n \quad (\text{by substitution})$$

$$2. \log \frac{m}{n} = \log m - \log n$$

$$\frac{m}{n} = \frac{10^x}{10^y} = 10^{x-y} \quad \text{or } \log \frac{m}{n} = x - y = \log m - \log n$$

$$3. \log m^p = p \log m$$

$$m^p = (10^x)^p = 10^{px} \quad \text{or } \log m^p = px = p \log m$$

$$\text{Hence } \sqrt[p]{m} = 10^{\frac{x}{p}} \quad \text{or } \log \sqrt[p]{m} = \frac{x}{p} = \log \frac{m}{p}$$

$$4. \log \sqrt[p]{m} = \log \frac{m}{p}$$

I. To multiply numbers.

Add their logarithms and find the antilogarithm of the sum. This will be the product of the numbers.

II. To divide one number by another number.

Subtract the logarithm of the divisor from the logarithm of the dividend and obtain the antilogarithm of the difference. This will be the quotient.

III. To raise a number to a required power.

Multiply the logarithm of the number by the index of the required power, and find the antilogarithm of the product.

IV. To extract the required root of a number.

Divide the logarithm of the number by the required root and find the antilogarithm of the quotient.

Illustrative Example 1. $561.75 \times .03286$

$$\begin{array}{rcl}
 \text{SOLUTION:} & \log 561.75 & = 2.7495 \\
 & \log .03286 & = 8.5167 - 10 \\
 & \hline
 & \log \text{ Product} & = 1.2662 \\
 & \text{Product} & = 18.45
 \end{array}$$

Illustrative Example 2. $46.72 \div .0998$

$$\begin{array}{rcl}
 \text{SOLUTION:} & \log 46.72 & = 1.6695 \\
 & & = 11.6695 - 10 \\
 & \log .0998 & = 8.9991 - 10 \\
 & \hline
 & \log \text{ Quotient} & = 2.6704 \\
 & \text{Quotient} & = 468.2
 \end{array}$$

Illustrative Example 3. Find the 6th power of .7929, or $.7929^6$

$$\begin{array}{rcl}
 \text{SOLUTION:} & \log .7929 & = 9.8992 - 10 \\
 & & \quad \quad \quad 6 \\
 & \hline
 & \log x & = 59.3952 - 60 \\
 & & = 9.3952 - 10 \\
 & x & = .2484
 \end{array}$$

Illustrative Example 4. Find the cube root of 532.768 or $\sqrt[3]{532.768}$.

$$\begin{array}{rcl}
 \text{SOLUTION:} & \log 532.768 & = 2.7265 \\
 & \frac{1}{3} \log 532.768 & = 0.9088 \\
 & x & = 8.1060
 \end{array}$$

WRITTEN EXERCISES

1. Multiply 763 by 298 by the use of logarithms, and check the result by actual multiplication.
2. Multiply 3.245 by 63.29.
3. Divide 19.65 by 2.843, and check by actual division.
4. Raise 17.64 to fourth power or find value of $(17.64)^4$.
5. Calculate the 5th root of 29.34 or find value of $\sqrt[5]{29.34}$.

Find value of the following:

- | | |
|---------------------------------------|---------------------------------------|
| 6. $26.45 \times .02687 \times 3.194$ | 11. 862×48.75 |
| 7. $336 \div 1984$ | 7.862×6.827 |
| 8. $527 \times .083$ | 12. .0734 raised to the fourth power |
| 9. 42.316 | 13. $.637^4$ |
| $.06214$ | 14. $89.76 \times 98.54 \times 26.63$ |
| 10. 1.78×19 | $.005862 \times 8271$ |
| 23.7 | 15. $.07^6$ |
| 16. Extract the fifth root of .0329. | |
| 17. Extract the fourth root of .0072. | |

Find the value of the following:

- | | |
|---|--|
| 18. $47.1 \times 3.56 \times .0079$ | |
| 19. $4.77 \times (-.71)$ | HINT: Work the same as if the sign were +, |
| $.83$ | and put in correct sign at end. |
| 20. -523×249 | 28. $79 \times 470 \times .982$ |
| 767×396 | 29. $8,075 \div 364.9$ |
| 21. $(1.032)^{15}$ | 30. $-.643 \times 7095$ |
| 22. $(.795)^6$ | $67 \times 9 \times -.462$ |
| 23. $(3.57)^4$ | 31. $(2.388)^5$ |
| 24. $\sqrt[7]{.00429}$ | 32. $(1.014)^{25}$ |
| 25. $\sqrt[2]{.005} \times \sqrt[3]{.0765}$ | 33. $.0325 \times .6425 \times 5.26$ |
| 26. $\sqrt[4]{10^3}$ | 34. The seventh root of .00898 |
| 27. $\sqrt[5]{529}$ | 35. $\sqrt[7]{15}$ |
| $\sqrt[6]{67 \times 518}$ | 36. 4.26 |
| | $7.42 \times .058$ |

37. Find the circumference of a circle whose diameter is 17.63 inches, from the equation $C_o = \pi d$. ($\pi = 3.1416$)

38. Find the area of an ellipse whose semi-axis a is 22.18 in. and shorter axis b is 16.88 in. from the equation, $A_{ellipse} = \pi ab$.

39. Find the area of a triangle whose sides are $a = 16.35$, $b = 18.97$, and $c = 24.77$ in. respectively, using the equation:

$$A_{\Delta} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}$$

40. The diameter of a spherical balloon which is to lift a given weight is calculated by the equation:

$$D = \sqrt[3]{\frac{W}{.5236(A-G)}} \quad \text{where}$$

D = diameter of balloon in feet

A = weight in pounds of a cu. ft. of air

G = " " " " " " " " gas in the balloon

W = weight to be raised (including the weight of the balloon)

Find D , if $A = .08072$; $G = .0056$; $W = 1,250$ lb.

41. Find W , using the equation of Exercise 40, if $A = .08072$; $G = .0056$; $D = 35.5$.

42. In warming a building by hot water pipes, the required length of pipes 4 in. in diameter is determined by the equation:

$$L = \frac{(P - t)(T - t)}{P - T} \times .0045 C \quad \text{where}$$

L = length of pipes in feet

P = temperature (Fahrenheit) of the pipes

T = " required in the building

t = " of external air

C = number of cu. ft. of space to be warmed

Find L when $P = 120^{\circ}$; $t = 40.5^{\circ}$; $T = 61.5^{\circ}$; $C = 35.6 \times 100$

43. If $8^x = 324$, find x .

SOLUTION:

$$\log 8^x = \log 324$$

$$x \log 8 = \log 324$$

$$x = \frac{\log 324}{\log 8}$$

$$= \frac{2.5105}{.9031}$$

$$= 2.75$$

44. If $3^x = 25$, find x .

45. If $64^x = 4$, find x .

46. If $16^x = 1,024$, find x .

47. If $2^x = 64$, find x .

CHAPTER XVI

COMMERCIAL APPLICATIONS OF LOGARITHMS

194. Calculation of Compound Interest.—

1. To find the amount when the principal, rate, and time are given. The amount at the end of one year is $p + pr$ or $p(1 + r)$, since p is the principal and pr is 1 year's interest. Thus to get the amount at the end of 1 yr. always multiply the principal by $1 +$ the rate.

Now, in compound interest the principal at the beginning of the second year is $p(1 + r)$. Then the amount at the end of the second year is $p(1 + r)(1 + r)$ or $p(1 + r)^2$; and so on for n years. Hence

$$A = p(1 + r)^n$$

Illustrative Example. Find the amount of \$725.15 at 5% compound interest compounded annually at the end of 6 yr. Also find compound interest.

SOLUTION:

$$\begin{aligned} A &= p(1 + r)^n \\ &= 725.15(1 + .05)^6 & \log 1.05 &= .0212 \\ \log A &= \log 725.15 + 6 \log 1.05 & 6 \log 1.05 &= .1272 \\ &= 2.8604 + .1272 \\ &= 2.9876 \\ A &= \$971.80 \\ Ci &= \$971.80 - \$725.15 = \$246.65 \end{aligned}$$

2. To find the cost or "present worth" of a sum payable n years hence, supposing interest to be compounded. By solving the above equation for p we get,

$$p = \frac{A}{(1 + r)^n}$$

Illustrative Example. Find the cost or present worth which will amount to \$923 at 4% compound interest in 12 yr.

SOLUTION

$$p = \frac{A}{(1+r)^n}$$

$$= \frac{923}{1.04^{12}}$$

$$\log p = \log 923 - 12 \log 1.04$$

3. To find the amount when interest is compounded q times a year, use the formula:

$$A = p \left(1 + \frac{r}{q}\right)^{qn}$$

WRITTEN EXERCISES

1. Find the amount of \$933 at 5% compounded annually for 7 yr.

2. Find the principal which amounts to \$775.20 in 15 yr. at 5% compounded annually.

3. Find the cost or present worth of \$918 to be paid in 10 yr. allowing 5% interest compounded annually.

4. Find the amount of \$700 which ran for 12 yr., interest being compounded semiannually at 6%.

$$A = p \left(1 + \frac{r}{2}\right)^{2n}$$

5. Find the amount of \$1 at 6% compound interest for 20 yr., compounded annually. Also find the compound interest.

6. Find the amount of \$1,250 for 12 yr. at 6% compounded annually.

7. Find the amount of \$25 for 500 yr. compounded annually at 5%.

8. Find the amount of \$300 for 50 yr. at 6% compounded annually.

9. Find the amount of \$300 for 50 yr. at 6% compounded semiannually.

10. Find the amount of \$300 for 50 yr. at 6% compounded quarterly.

11. In how many years will \$1 double itself at 3% interest compounded annually?

SOLUTION:

$$\begin{aligned}
 1.03^x &= 2 \\
 x \log 1.03 &= \log 2 \\
 x &= \frac{\log 2}{\log 1.03} \\
 &= \frac{.30103}{.01284} \\
 &= 23.5 \text{ years}
 \end{aligned}$$

12. In how many years will \$1 double itself at 5% compounded annually? At 4%? At 6%?

13. In how many years will \$4,000 amount to \$7,360.80 at 5% interest compounded annually?

14. In how many years will \$12 double itself at 4% interest compounded semiannually?

SOLUTION:

$$\begin{aligned}
 A &= P (1 + \frac{r}{2})^{2n} \\
 24 &= 12 (1 + .02)^{2n} \\
 2 &= 1.02^{2n} \\
 2n \log 1.02 &= \log 2 \\
 2n &= \frac{\log 2}{\log 1.02} \\
 &= \frac{.30103}{.00860} \\
 &= 35 \\
 n &= 17.5 \text{ yr.}
 \end{aligned}$$

15. In how many years will \$100 double itself at 6% interest compounded semiannually?

16. In how many years will \$100 double itself at 6% interest compounded quarterly?

17. What sum of money will amount to \$400 in 10 yr. if placed at interest at 4%, if compounded annually?

195. Sinking Fund Calculations.—(See also § 43). The following explains the application of logarithms to sinking fund calculations.

If the sum set apart at the end of each year to be put at compound interest is represented by S , and

P	= principal
r	= interest on \$1 for 1 yr.
$R = (1 + r)$	= amount of \$1 for 1 yr.
A	= amount of P for n yr.

then the sum at the end of the

$$\begin{array}{ll}
 \text{1st yr.} & = S \\
 \text{2d " } & = S + SR \\
 \text{3d " } & = S + SR + SR^2 \\
 \text{\textit{n}th " } & = S + SR + SR^2 + \dots + SR^{n-1} \\
 \text{That is } A & = S + SR + SR^2 + \dots + SR^{n-1} \quad (1) \\
 \therefore AR & = SR + SR^2 + SR^3 + \dots + SR^n \quad (2) R \times (1) \\
 \therefore AR - A & = SR^n - S \quad (3) (2) - (1) \\
 A(R - 1) & = S(R^n - 1) \quad (4) \text{Factor left member} \\
 & \quad \text{and right member} \\
 A & = \frac{S(R^n - 1)}{R - 1} \quad (5) \div \text{by } R - 1 \\
 & = \frac{S(R^n - 1)}{r} \quad (6) R - 1 = ?
 \end{array}$$

(Note that (1) above is a geometric progression.)

Illustrative Example 1. If \$10,000 be set apart annually, and put at 6% compound interest for 10 yr., what will be the amount?

SOLUTION:

$$\begin{aligned}
 A &= \frac{S(R^n - 1)}{r} \\
 &= \frac{10,000(1.06^{10} - 1)}{.06} \\
 &= \$131,808 + \text{(with a 6-place table)}
 \end{aligned}$$

Illustrative Example 2. A county owes \$60,000. What sum must be set apart annually, as a sinking fund, to cancel the debt in 10 yr., provided money is worth 6%? Find total cost each year.

SOLUTION:

$$\begin{aligned}
 S &= \frac{Ar}{R^n - 1} \\
 &= \frac{\$60,000(.06)}{(1.06)^{10} - 1} \\
 &= \frac{3,600}{1.79085 - 1}
 \end{aligned}$$

$$\begin{aligned}
 &= \$4,552. + \quad (6\text{-place table}) \\
 \text{Yearly interest} &= 6\% \text{ of } \$60,000 \\
 &= \$3,600 \\
 \text{Total cost} &= \$3,600 + \$4,552 \\
 &= \$8,152
 \end{aligned}$$

WRITTEN EXERCISES

1. Find the amount of \$20,000 set apart annually and put at 5% compound interest¹ for 20 yr.

2. A city is bonded for \$50,000. What sum must be set aside annually as a sinking fund, to cancel the debt in 20 yr., provided money is worth 5%?

3. If an annual life insurance premium is \$150 and money is worth 4%, what is the value of the sum of the premiums at the end of 20 yr.?

196. Annuities.—An annuity is a sum of money that is payable yearly, or in parts at fixed periods in the year.

197. Finding the Amount of an Unpaid Annuity.—To find this amount when the interest, time, and rate per cent are given, we let the sum due at the end of the

$$\begin{aligned}
 \text{1st yr.} &= S \\
 \text{2d " } &= \text{etc., as in § 194} \\
 \text{That is } A &= \frac{S(R^n - 1)}{r}
 \end{aligned}$$

Illustrative Example. An annuity of \$1,200 was unpaid for 6 yr. What was the amount due if interest is reckoned at 6%?

SOLUTION:

$$\begin{aligned}
 A &= \frac{S(R^n - 1)}{r} \\
 &= \frac{\$1,200(1.06^6 - 1)}{.06} \\
 &= \$8,360 +
 \end{aligned}$$

¹ Unless otherwise stated, interest is compounded annually.

WRITTEN EXERCISES

1. An annual pension of \$600 was unpaid for 5 yr. Find the amount due if interest is computed at 5%.

2. A widow receives a pension of \$100 annually from the United States government and back-pay for 8 yr. What should she receive in back-pay if interest is at 3% per year?

198. Finding the Present Value of an Annuity.—To find the present value of an annuity, when the time it is to continue and the rate per cent are given, we use the following formula:

P = present value

A = amount of P for n years, or the amount of the annuity for n years

But the amount of P for n years

$$= P (1 + r)^n$$

$$= PR^n$$

And $A = \frac{S (R^n - 1)}{R - 1}$ = amount of the unpaid annuity for n years

$$\text{Hence } PR^n = \frac{S (R^n - 1)}{R - 1} \quad \text{Since } A = PR^n$$

$$\therefore P = \frac{S (R^n - 1)}{R^n (R - 1)}$$

$$P = \frac{S}{R-1} \times \frac{R^n-1}{R^n}$$

If the annuity is perpetual, the fraction $\frac{R^n - 1}{R^n}$ approaches 1 as its limit.

$$\therefore P = \frac{S}{r} \quad (\text{when annuity is perpetual})$$

Illustrative Example 1. Find the present value of an annual pension of \$1,000 for 5 yr., at 4% interest.

SOLUTION:

$$\begin{aligned}
 P &= \frac{S}{R^n} \times \frac{R^n - 1}{R - 1} \\
 &= \frac{1,000}{1.04^5} \times \frac{(1.04^5 - 1)}{1.04 - 1} \\
 &= \frac{1,000}{1.21661} \times \frac{1.21661 - 1}{.04} \\
 &= \frac{216.61}{.0486644} \\
 &= \$4,451 +
 \end{aligned}$$

Illustrative Example 2. Find the present value of a perpetual scholarship that pays \$300 annually, at 6% interest.

SOLUTION:

$$\begin{aligned}
 P &= \frac{S}{r} \\
 &= \frac{300}{.06} \\
 &= \$5,000
 \end{aligned}$$

WRITTEN EXERCISES

1. Find the present value of an annual pension of \$1,200 to continue 12 yr., at 4% interest.

2. A man is retired by a railroad on a yearly pension of \$900. He lives 9 yr. and 6 mo. What is the value of such pension if money is worth 4½%?

3. Find the present value of a perpetual scholarship of \$450 per year at 5%.

4. Find the present value of a property purchased on a basis of \$500 paid annually for 15 yr., if money is worth 4%.

199. Finding Present Value of Annuities.—The present value of a perpetual annuity which shall begin in a given number of years, when the time it is to continue and the rate per cent are given, may be found by the following formula:

$$P = \frac{S}{R^p (R - 1)} \quad (\text{where } p = \text{number of years before annuity begins})$$

Illustrative Example 1. Find the present value of a **perpetual** annuity of \$1,000, to begin in 3 yr., at 4% interest.

SOLUTION:

$$\begin{aligned} P &= \frac{S}{R^p (R - 1)} \\ &= \frac{\$1,000}{(1.04)^3 \times .04} \\ &= \$22,225 \end{aligned}$$

Illustrative Example 2. Find the present value of a **term** annuity of \$5,000, to begin in 6 yr., and to continue 12 yr. at 6%.

SOLUTION:

$$\begin{aligned} P &= \frac{S}{R^p + q} \times \frac{R^q - 1}{R - 1} \quad (\text{where } q = \text{number of years that} \\ &= \frac{\$5,000}{1.06^{18}} \times \frac{(1.06)^{12} - 1}{.06} \quad \text{annuity is to continue}) \\ &= \$29,550 \end{aligned}$$

WRITTEN EXERCISES

1. Find the present value of a perpetual annuity of \$500, to begin in 8 yr. at 4% interest.
2. Find the present value of an annuity of \$1,200, to begin in 10 yr. and continue for 15 yr. at 5%.
3. A man is 55 yr. of age. He is to be retired at 70 on an annual pension of \$900. Suppose that he lives until he is 85. Find the present value of such a pension at 4% interest.

200. Finding the Annuity.—To find this when the present value, the time, and the rate per cent are given, the following formula may be applied:

$$P = \frac{S (R^n - 1)}{R^n (R - 1)}$$

$$\begin{aligned}\therefore S &= \frac{PR^n(R-1)}{R^n-1} \\ &= Pr \times \frac{R^n}{R^n-1}\end{aligned}$$

Illustrative Example. What annuity for 5 yr. will \$4,675 give when interest is reckoned at 4%?

SOLUTION:

$$\begin{aligned}S &= Pr \times \frac{R^n}{R^n-1} \\ &= \$4,675 \times .04 \times \frac{1.04^5}{(1.04)^5-1} \\ &= \$1,050\end{aligned}$$

WRITTEN EXERCISES

1. What annuity will \$6,000 buy for 10 yr. if interest is reckoned at $3\frac{1}{2}\%$?
2. What annual pension will \$10,000 buy for 20 yr. if money is worth 4%?
3. \$4,000 will buy what annuity for 10 yr. at 4%?

201. Life Insurance.—In order that a certain sum may be secured, to be payable at the death of a person, he pays yearly a fixed premium, according to the following formulas:

$$\begin{aligned}P &= \text{premium to be paid for } n \text{ years} \\ A &= \text{amount to be paid immediately after the last premium} \\ A &= \frac{P(R^n-1)}{R-1} \\ \therefore P &= \frac{A(R-1)}{R^n-1} \\ P &= \frac{Ar}{R^n-1}\end{aligned}$$

If A is to be paid 1 yr. after the last premium then

$$P = \frac{A(R-1)}{R(R^n-1)}$$

$$P = \frac{Ar}{R(R^n-1)}$$

To find the number of years the premium should be paid, in order that the company shall sustain no loss, the following formula may be used:

$$R^n = 1 + \frac{Ar}{S} \text{ or } n = \frac{\log \left(1 + \frac{Ar}{S}\right)}{\log R}$$

In the calculation of life insurance it is necessary to employ tables which shall show for any age the probable duration of life.

The following table gives the number of survivors at the different ages out of 100,000 persons alive at the age of 10.

AGE	SURVIVORS	AGE	SURVIVORS
10	100,000	55	64,563
15	96,285	60	57,917
20	92,637	65	49,341
25	89,032	70	38,569
30	85,441	75	26,237
35	81,822	80	14,474
40	78,106	85	5,485
45	74,173	90	847
50	69,804	95	3

Illustrative Example. Taking the figures of the above table, calculate what the chance is that a person 15 yr. of age will live to the age of 35?

SOLUTION:

$$\frac{81,822}{96,285}$$

ORAL AND WRITTEN EXERCISES

1. What is the chance that a person 40 yr. old will live to be 80 yr. old? That a person 70 yr. old will live to be 90 yr. old?
2. If a person now is 20 yr. of age, what are the chances that he will live to be 45? To be 50? To be 65? To be 80?
3. What annual premium should be charged for a policy worth \$1,000 at the end of 20 yr. if money is worth 4%?
4. If the annual premium is \$50, the amount of the policy is \$2,000, and money is worth 4%, for how many years must the premium be paid that no loss shall be sustained by the company?
5. What annual premium should be charged for a policy worth \$2,000 at the end of 10 yr., if money is worth 4%?

MISCELLANEOUS WRITTEN EXERCISES

Use logarithms in solving the following:

1. To what will \$3,750 amount in 20 yr. at 5% compounded annually?
2. To what does \$1,000 amount in 10 yr. if left at 4% compounded: (a) Annually? (b) Semiannually? (c) Quarterly?
3. A sum of money left at 4½% compounded annually for 30 yr. amounts to \$30,000. What is the sum?
4. At what per cent interest must \$15,000 be left in order to amount to \$60,000 in 32 yr. compounded annually?
5. At what per cent must \$3,333 be left so that in 24 yr. it will amount to \$10,000 compounded annually?
6. In how many years will a sum double itself if left at 6% interest compounded annually?
7. Find the amount of \$2,500 in 18 yr. at 4% compounded annually. Find also the compound interest.
8. What sum should be paid for an annual pension of \$1,000 payable annually for 20 yr., money being worth 3% per annum compound interest if

$$\text{Present value} = \frac{A}{r} \left(1 - \frac{1}{(1 + r)^n} \right)$$

9. What sum will amount to \$1,250 if put at compound interest at 4% for 15 yr.?
10. If \$1,600 is placed at 3½% interest semiannually for 13 yr., to how much will it amount in that time?

11. A person borrows \$600. How much must he pay annually that the whole debt may be paid in 35 yr., allowing interest at 4% compounded annually?

12. Find the amount of \$100 in 25 yr., at 5% per annum, compounded annually.

13. What is the present worth of \$1,000 payable at the end of 100 yr., interest being at the rate of 5% per annum and compounded annually?

14. Find the present value of an annuity of \$100 to be paid for 30 yr., reckoning interest at 4% compounded annually.

15. Find the amount of \$1 in 100 yr. at 5% compound interest compounded annually.

16. Find the amount of \$500 in 10 yr. at 4% compounded semi-annually.

17. What is the present value of \$1,000 which is to be paid at the end of 15 yr., reckoning interest at 3% compounded annually?

18. What is the present value of an annuity of \$500 that ceases at the end of 25 yr., interest reckoned at 6%?

19. If the population of a state increases in 10 yr. from 2,009,000 to 2,487,000, find the average yearly rate of increase if

$$\text{Average rate} = R - 1, \text{ and} \\ R^n = \frac{\text{population at end}}{\text{population at beginning}} \text{ or } R = \sqrt[n]{\frac{P_e}{P_b}}$$

20. If the population of a state now is 1,918,600 and the yearly rate of increase is 2.38%, find the population after 10 yr. hence if

$$\text{Population at end} = P_b (\text{yearly rate} + 1)^n \\ P_b = \text{population at beginning}$$

21. A man borrows a sum of money at 3½% interest annually, and lends the same at 5% quarterly. If his annual gain is \$441, find the sum borrowed.

22. If the annual premium is \$150, the amount of the policy is \$5,000, and money is worth 4%, for how many years must the premium be paid that no loss shall be sustained by the company?

23. If a city wishes to take up \$2,500,000 worth of bonds at the end of 4 yr., how much must it set aside each year, if the rate of interest is 5% and

$$C = \frac{S [(1 + r)^n - 1]}{r} \text{ where } \begin{cases} C = \text{number of dollars in debt} \\ n = \text{number of years} \\ S = \text{sum set aside annually} \\ r = \text{rate of interest} \end{cases}$$

24. Find the amount of \$5,000 at the end of 10 yr., interest at 8% compounded annually.

25. A sum of money is left 22 yr. at 4% compounded annually and amounts to \$17,000. Find the principal which was originally put at interest.

26. Find the amount and the compound interest on \$1,000 at 4% for 10 yr. compounded annually; then find the amount and compound interest on the same for 4 yr. at 10% compounded annually; then find the difference between the two results. Which is the greater?

27. What sum should be paid for an annuity of \$1,200 a year to be paid for 30 yr., money being worth 4% compounded annually?

28. A premium of \$120 is paid each year for 10 yr. Find the value of the sum of these premiums at the end of the 10th yr., with interest at 4% compounded annually, if

$$\text{Value} = \text{Premium} \times R \left(\frac{R^n - 1}{R - 1} \right)$$

29. A man invests \$200 a year in a savings bank which pays $3\frac{1}{2}\%$ per annum on all deposits. What will be the total amount due him at the end of 25 yr.?

30. Twenty annual payments of \$500 each are deposited with an assurance company for the benefit of a person to whom, beginning with the 20th yr., the entire amount paid in, together with accruing interest, is to be returned in 40 equal annual payments. Reckoning interest at 5%, what should be the amount of each payment?

31. The sum of \$100 was deposited in a bank at compound interest on Jan. 2 every year for 10 yr. At the beginning of the 11th yr. and on each succeeding Jan. 2 during 10 yr., \$100 was withdrawn. Interest being reckoned at 5%, what amount remained on deposit Jan. 1 at the end of the 10th yr. of withdrawals?

32. If the average death rate per annum in a city be $1\frac{7}{8}\%$ and the average birth rate be $2\frac{2}{3}\%$, and if there be no increase or decrease in the population by migration, in how many years will the population be doubled?

33. A man borrows \$6,000 to build a house, agreeing to pay \$50 monthly until the principal, together with interest at 6% is paid. Find the number of full payments required.

34. If each payment in Exercise 33 is at once loaned at 6%, compounded annually, what will they all amount to by the time the final payment of \$50 is made?

35. From Exercises 33 and 34 determine the total interest received by the money lender up to the time of the last payment. What per cent on the original \$6,000 is this?

202. Bonds.—To find what interest on his investment a purchaser will receive, the following formulas may be used:

P = price of a bond that has n years to run

r = per cent it bears

S = face of bond (usually \$100 or \$1,000)

q = current rate of interest

Let x = rate of interest on the investment

Then, $P(1+x)^n$ = value of purchase money at the end of n years

$Sr(1+q)^{n-1} + Sr(1+q)^{n-2} + \dots + Sr + S$ = amount of money received
bond if interest on bond
put immediately at co
pound interest at $q\%$

$$\begin{aligned} \text{But, } Sr(1+q)^{n-1} + Sr(1+q)^{n-2} + \dots + Sr + S &= S + \frac{Sr[(1+q)^n - 1]}{q} \\ P(1+x)^n &= S + \frac{Sr[(1+q)^n - 1]}{q} \\ 1+x &= \left(\frac{S}{P} + \frac{Sr[(1+q)^n - 1]}{Pq} \right) \\ 1+x &= \left(\frac{Sq + Sr(1+q)^n - Sr}{Pq} \right) \end{aligned}$$

Illustrative Example 1. What is the rate of interest on a 4% bond at 114, that has 26 yr. to run, if money is worth $3\frac{1}{2}\%$?

$$\begin{aligned} 1+x &= \left(\frac{3.5 + 4(1.035)^{26} - 4}{114 \times .035} \right) \frac{1}{26} \\ 1+x &= 1.033 \\ x &= .033 \end{aligned}$$

$\therefore$ Purchaser receives 3.33%.

Illustrative Example 2. At what price must 7% bonds be bought, running 12 yr., with the interest payable semiannually, in order that the purchaser may receive on his investment 5% interest semiannually?

$$\begin{aligned} P(1+x)^n &= \frac{Sq + Sr(1+q)^n - Sr}{q} & S &= 100 \\ & & q &= .025 \text{ (interest semiannual-} \\ & & & \text{ly)} \\ P &= \frac{Sq + Sr(1+q)^n - Sr}{q(1+x)^n} & r &= .035 \\ &= \frac{2.5 + 3.5(1.025)^{24} - 3.5}{.025(1.025)^{24}} & n &= 24 \\ & & x &= .025 \\ &= 118 \end{aligned}$$

WRITTEN EXERCISES

1. If \$126 is paid for bonds due in 12 yr. and yielding $3\frac{1}{2}\%$ semiannually, what per cent is realized on the investment, provided money is worth 2% semiannually?

2. When money is worth 2% semiannually, if bonds having 12 yr. to run and bearing semiannual coupons of $3\frac{1}{2}\%$ each are bought at 114 $\frac{1}{8}$, what per cent is realized on the investment?

3. What may be paid for bonds due in 10 yr., and bearing semiannual coupons of 4% each, in order to realize 3% semiannually, if money is worth 3% semiannually, when

$$\frac{P}{S} = \frac{q + r(1 + q)^n - r}{q(1 + x)^n}$$

4. If $4\frac{1}{4}\%$ Liberty bonds maturing in 30 yr. are bought at 96.76 and money is worth 4% , what is the yield?

5. When $4\frac{1}{4}\%$ United States Third maturing in 10 yr. are bought at 97.16 and money is worth 6% , what is the yield?

6. What may be paid for bonds due in 25 yr., and bearing semiannual coupons of 4% each, in order to realize $4\frac{1}{2}\%$ semiannually if money is worth 3% semiannually?

7. What is the yield on New York City $4\frac{1}{2}\%$'s due in 45 yr. at 102 $\frac{1}{4}$ if money is worth 4% ?

CHAPTER XVII

THE SLIDE RULE

203. History and Use.—So far as has been determined, the slide rule as an instrument having one piece arranged to slide along another, was invented, according to Cajori, by William Oughtred between 1620 and 1630. The present arrangement of scales (see Form 19) was devised by Lieutenant Mannheim of the French army about 1850.

It was originated undoubtedly because of the fact that it is a time and labor saver. It should be borne in mind that in nearly all practical calculations only an approximately correct answer is necessary, and the skill of the operator is often best shown by his ability to approximate to the right degree of accuracy. If the result is as accurate as the data employed to obtain it, or as accurate as our answer is required to be, then we have accomplished an economy of time and labor.

It is entirely possible, after having attained proficiency in handling the slide rule, to obtain results which shall not have more than $\frac{1}{2}$ of 1% of error. This is perfectly satisfactory for many of the problems of the business world.

The slide rule is used in the office, either to check figures, or for original calculation. It computes mensuration, payroll, interest, percentage rates, discount, profit and loss, foreign exchange, freight, prorating, compound interest, and has many other applications of the kind in the business field.

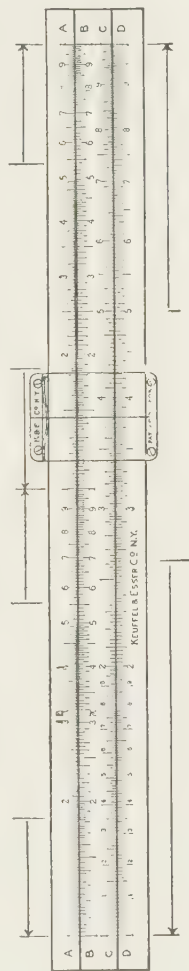
The purpose of this chapter is to explain the use of this little computing stick so that you can apply it to your own particular kind of work.

204. Description of the Slide Rule.—

Since we found in the study of logarithms that we could multiply numbers by adding their logarithms, we also have found out that we can add lengths or logarithms on a rule and oftentimes simplify our work. This is done by the use of two rulers which slide along each other. The rulers are marked to show logarithms of numbers, and by adding these logarithms we can easily find the logarithm of their product, and then the product. We can also use the slide rule to divide, to find the roots and powers of numbers. Each number printed on the slide rule stands in the position indicated by its logarithm.

In Form 19, BC is the slide, graduated on the upper and lower edges. These graduations were made as follows: CC was divided into 1,000 equal parts; $\log 2 = .301$, therefore 2 was placed at the 301st graduation; $\log 3 = .477$, therefore 3 was placed at the 477th graduation; and so on for all the integers from 1 to 1,000.

In order to read the numbers from 1 to 1,000, we go over the rule from left to right. We read first 1, 2 . . . 10; then



Form 19. The Slide Rule

beginning at 1 again and calling it 10, read it 10, 20 . . . 100; then beginning at 1 again read it 100, 200 . . . 1,000. This is allowable because the mantissa for 10 is the same as that for 100, 1,000, etc. It will be noted that there is a decrease in the lengths of the spaces from left to right. These decreases in lengths correspond exactly to the differences between the logarithms from 1 to 10. We can also put in marks to show the mantissas for the logarithms of 1.5, 2.5, etc. Now since $\log 1.5$ is 0.176, this is not half the difference between $\log 1$ and $\log 2$, therefore the mark does not exactly bisect the line from 1 to 2

205. How to Read the Slide Rule.—On BB it will be noted that the distance from 1 to 2 is divided into what we shall call 10 large divisions and they will be read from 1 at the left (toward the right) as follows: 11, 12 . . . 19, 2 (read as telephone numbers one-one, one-two, etc., or understood as 1.1, etc.). It will also be noted that each of these large divisions is again divided into 5 parts, each of which denotes .02, so that the second division after the first mark of the large division would be read 1.14, and the fourth small division after the mark denoting the fifth large division would be read 1.58, etc. It will be noted that the number of large divisions from 2 to 3 is also 10, but that each large division is subdivided again into only 2 small divisions, so that the small mark after the first large division between 2 and 3 would be read 2.15, etc. The same scheme works from 3 to 5. From 5 to 6 it will be observed that there are but 2 large divisions and 5 small ones in each of the large divisions. The first large mark there would be read 5.5, and the first small mark following this large mark would be read 5.6. The divisions are the same from 6 to 10. If the little runner

having a hair line on it, which you find on the ruler, should be used and the hair line should fall half way between the first small line after 5 and the second small line after 5, it would be read 5.15, etc. The same plan of ruling will be found on the right-half of the ruler commencing with the second 1 marked on the ruler.

206. Operations with the Slide Rule.—It is not difficult to learn to use the slide rule if the student will use small numbers at first. If in doubt how to do an operation, try it first with small numbers which you can easily check mentally.

1. *Multiplication.* Multiply 2 by 4. Move the slide (the part of the rule in the middle which slides) so as to set the 1 of the B scale directly under 2 of the A scale, and read the answer 8 on the A scale directly above the 4 of the B scale; or set the 1 of the C scale directly above the 2 of the D scale and read the answer 8 on the D scale directly below the 4 of the C scale. Hence, to find the product of two numbers, set the 1 of the C scale on one of the numbers on the D scale, and under the other number on the C scale read the product on the D scale.

Sometimes in multiplying we will have to use the 1 at the right-hand end of the C scale. For example, multiply 86 by 2. Set 1 at the right-hand end of the C scale on 86 of D, and under 2 of C read the product 172 on D. We simply use the 1 at the left end or the 1 at the right end of C, according as it brings the other number over scale D. It will be observed in the above example that if we had used the 1 on the left end of C, it would have brought the 2 of C off the scale D.

Place your decimal point by inspection. Thus to multiply 10.5 by 1.8, set 1 C on 18 D, and under 105 C read the number 189 on D. Then make an approximate multiplica-

tion mentally, $10 \times 2 = 20$; hence we know that there are two integral figures in the product, giving 18.9 as the result. The decimal point will have to be placed by making an approximate calculation mentally.

2. *Division.* Divide 6 by 2. Set 2 C' over 6 D, and read the result directly under 1 C' on D. Therefore to divide one number by another, set the divisor on scale C' over the dividend on scale D, and under 1 C' read the quotient on scale D. Here again the decimal point is placed by inspection. Thus to divide 2.85 by 15, set 15 C' over 285 D, and under 1 C' read the quotient 19 on D; but we can observe that $3 \div 15$ is about $\frac{3}{15}$, or $\frac{1}{5}$ or .2; hence our quotient is .19.

3. *Combined Multiplication and Division.* Find the value of $\frac{26 \times 4}{8}$. Set 8 C' over 26 D, and under 4 C' read the result 13 on D. First the division of 26 by 8 is made by setting 8 C' over 26 D, and under 1 C' we might read the quotient; but we want to multiply this quotient by 4. As 1 C' is already on this quotient we have only to read the product 13 on scale D under 4 C'. By the use of this scheme we can find the fourth term of a proportion. For example, in the proportion $8:26 = 4:x$, $x = \frac{26 \times 4}{8}$. Therefore to find the fourth term of a proportion, set the first term over the second, and under the third read the fourth term.

4. *Continued Multiplication and Division.* In this work use the little glass (or celluloid) runner which has the hair line on it.

Illustrative Example 1. Find the value of $4 \times 6 \times 3$.

SOLUTION: Set 1 C' at the right over 4 D, set the runner on 6 C', set 1 C' at the right on the runner (sliding glass), under 3 C' read 72 on D.
 $\therefore 4 \times 6 \times 3 = 72$.

Illustrative Example 2. Find the value of $\frac{72}{4 \times 9}$.

SOLUTION: Set 4 C over 72 D, runner on 1 C, set 9 C on runner, under 1 C read the result 2 on D.

Illustrative Example 3. Find the value of $\frac{16 \times 36}{12 \times 8}$.

SOLUTION: Set 12 C over 16 D, set runner on 36, set 8 C on runner, and under 1 C read the result 6.

Illustrative Example 4. Find the value of $\frac{4 \times 3 \times 8}{16}$.

SOLUTION: Set 16 C over 4 D, set runner on 1 C, set 1 C at the right end of the slide on the runner, set runner on 3 C, set 1 C on runner, under 8 C read the result 6 on D. We can work any continued multiplications and divisions in a similar manner.

5. *Squares and Square Root.* Note that the graduations on the upper scale A are the squares of the numbers directly below on scale D. For example, the square of 2 is 4, and above 3 is 9, above 6 is 36, above 15 is 225. The first 4 on A is either 4 or 400, the square of either 2 or 20 respectively on scale D. The second 4 on A is either 40 or 4000, the square of 6.32 or 63.2 of D. Hence, to square any number, find the number on scale D and read its square directly above it on scale A. To find the square root of any number, find the number on scale A and read its square root directly below it on scale D.

Scale A will be found very useful when dividing or multiplying by square roots, finding area of circles, etc.

Illustrative Example 1. Find the value of $5\sqrt{2}$.

SOLUTION: Set 1 C at left end of scale on 2 A, under 5 C read the result 7.07 on D.

Illustrative Example 2. Find the value of $\frac{5}{\sqrt{2}}$.

SOLUTION:

$$\frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

Set 2 C on 2 A, and under 5 C read the result 3.53 + on D.

Illustrative Example 3. Find the value of $\frac{\sqrt{6} \times \sqrt{14}}{\sqrt{7}}$.

SOLUTION: Set 7 B on 6 A, and under 14 B read the result 3.47 on D.

Illustrative Example 4. Find the area of a circle whose radius is 3 inches.

SOLUTION: Set 1 C on 3 D, and above π on B read the area, 28.27 sq. in. on A.

NOTE: If the student can do his work on the slide rule so that it is correct to the first decimal, this will be satisfactory for most computations.

EXERCISES

Find the value of the following:

- | | |
|-------------------------|---|
| 1. 65×4 | 22. $\frac{6.25}{2.5}$ |
| 2. 4.6×3.5 | 23. $38 \div 18$ |
| 3. 7.2×5.54 | 24. $2.5 \div .5$ |
| 4. 10.5×22.8 | 25. $4 \div 8$ |
| 5. $.08 \times 2.6$ | 26. $55 \div 27$ |
| 6. $.28 \times .004$ | 27. $31.25 \div 25$ |
| 7. $.54 \times 1.8$ | 28. $\frac{3 \times 4 \times 12}{2 \times 6}$ |
| 8. 2.6×18.5 | 29. $\frac{5 \times 7 \times 36}{6 \times 14}$ |
| 9. 4.4×18.4 | 30. $\frac{35 \times 64 \times 8}{7 \times 16 \times 4}$ |
| 10. $.54 \times .92$ | 31. $\frac{49 \times 54 \times 9}{18 \times 27 \times 7}$ |
| 11. $4.84 \times .005$ | 32. $\frac{10 \times 35 \times 65}{3 \times 8.4 \times 6.6}$ |
| 12. $.128 \times 64$ | 33. $\frac{4 \times 4.6 \times 2.6}{16.4 \times 12 \times 4.2}$ |
| 13. $45.2 \div 25$ | 34. $\frac{2.6 \times 8.4}{3 \times \sqrt{6}}$ |
| 14. $144 \div 24$ | 35. $7 \div \sqrt{6}$ |
| 15. $8.84 \div .75$ | 36. $\frac{\sqrt{8} \times \sqrt{10}}{\sqrt{3}}$ |
| 16. $128 \div 4.4$ | |
| 17. $\frac{26.8}{4.6}$ | |
| 18. $\frac{4.28}{.65}$ | |
| 19. $\frac{17.28}{1.2}$ | |
| 20. $\frac{625}{25}$ | |
| 21. $\frac{6.25}{.25}$ | |

38. Find the area of a rectangle whose length is 4.6 in. and whose width is 2.8 in.

39. Find the area of a circle whose radius is (a) 4 in.; (b) 2.8 in.; (c) 6.2 ft.; (d) 9.6 yd.

40. Find the circumference of a circle whose radius is 6.4 ft. if the circumference equals twice the radius times π .

41. Find the area of a lateral cylinder whose radius is 3 in. and whose height is 6.4 in.

42. If the wages of 6 men for 1 da. are \$28.50, what are the wages of 12 men at the same rate?

43. A department store offered a sale of 7 articles for 47¢, find the cost of 13 articles at the same rate.

44. 8 is what per cent of 24?

HINT: $8 \div 24 = \text{what decimal} = \text{what } \%$

45. $\frac{5}{16} = \text{what } \%$

46. $\frac{7}{24} = \text{what } \%$

47. $\frac{17}{32} = \text{what } \%$

48. $\frac{9}{64} = \text{what decimal?}$

49. $\frac{9}{72} = \text{what decimal?}$

50. Find the interest on \$800 at 5% for 27 da.

HINT:

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time in days}}{360}$$

51. Compute the interest on \$650 at 4% for 65 da.

52. What is the interest on \$500 for 75 da. at $4\frac{1}{2}\%$?

53. Find the principal which will produce \$120 in 5 yr. at 6%.

HINT:

$$\text{Principal} = \frac{\text{Interest}}{\text{Rate} \times \text{Time (in years)}}$$

54. Find the principal which will yield \$2,400 in 8 yr. at $5\frac{1}{2}\%$.

55. Find the square of each of the following:

- | | |
|----------|----------|
| (a) 4.6 | (f) 2.56 |
| (b) 6.8 | (g) 25.6 |
| (c) 12 | (h) .15 |
| (d) 9 | (i) 1.5 |
| (e) 10.6 | (j) 15 |

56. Find the square root of each of the following:

- | | |
|---------|----------|
| (a) 64 | (e) 1.44 |
| (b) 49 | (f) 14.4 |
| (c) 9 | (g) 2 |
| (d) 144 | (h) 3 |

NOTE: Check (g) and (h) by extracting their square roots, then memorize the result correct to 3 decimals.

57. Find the value of each of the following:

- | | |
|----------------------|--------------------------|
| (a) $\pi \times 12$ | (d) $\pi \times (5.4)^2$ |
| (b) $\pi \times 6$ | (e) $\pi \times (4.6)^2$ |
| (c) $\pi \times 4^2$ | (f) $\pi \times (2.4)^2$ |

58. Find the radius of a circle whose area is 144 sq. in.

59. Find the radius of a circle whose circumference is 31.416.

60. Find the circumference of a circle whose area is 78.54 sq. in.

207. To Find Cubes and Cube Roots.—To find the cube of 4, work as follows: Set 1 of B over 4 of D and read the answer 64 on A directly over 4 of B.

To find the cube root, reverse the process. For example, to find the cube root of 64, move the slide back and forth until the number on B directly under 64 (on A) is the same as that under 1 C, on D. If the left-hand 1 does not work, use the right-hand 1 on C.

EXERCISES

1. Find the cube of the following numbers:

- | | |
|---------|-----------|
| (a) 2 | (f) 6.4 |
| (b) 3 | (g) 1.2 |
| (c) 5 | (h) 1.25 |
| (d) 15 | (i) .04 |
| (e) 4.2 | (j) .0015 |

2. Find the approximate cube roots of the following:

- | | |
|---------|----------|
| (a) 8 | (e) 68 |
| (b) 27 | (f) 425 |
| (c) 125 | (g) 16.8 |
| (d) 216 | (h) 2.64 |

3. 6 times the cube root of 16 = ?
4. 4 times the cube of 1.6 = ?
5. Divide 18 by the cube root of 6.
6. Divide the cube root of 6 by 4.

MISCELLANEOUS EXERCISES

1. To find the 4th term of a proportion.

HINT: If $\frac{a}{b} = \frac{c}{d}$, then $d = c \times \frac{b}{a}$.

Set the first term on C to the second term on D, run the rider to the third term on C, and under the rider find the fourth term on D.

2. Find the fourth term of the proportion $6 : 18 = 7 : x$.
3. To find the mean proportional between two given numbers.

HINT: The proportion is $\frac{a}{x} = \frac{x}{c}$, or $x = \sqrt{ac}$.

Set index of scale B to a on scale A, and place the rider opposite c on scale B, then under the rider on D scale find the mean proportional required.

4. Find the mean proportional between 27 and 13.
5. To reduce to the decimal of a given quantity.
Express 4 oz. 10 dr. as a decimal of 1 ton.

Our fraction is $\frac{74}{16 \times 16 \times 2,000} 4 \text{ oz. } 10 \text{ dr.} = 74 \text{ dr.}$

HINT: Work out the denominator first, then the resulting fraction.

6. To find the interest on a sum of money.
Find the interest on \$500 at 4% for 4.5 yr.

HINT: $I = \frac{Pnr}{100}$

Railway apportionment of fares for different roads.

7. Suppose a fare of \$10 has been paid and the traveler goes over three different roads, making 250, 140, and 110 miles respectively on the different roads, find the amounts that should be apportioned to the different roads.

HINT: $\frac{250}{600} = \frac{1}{2}$ for the first road.
 $\frac{140}{600} = \frac{14}{60} = \frac{7}{30}$ for the second road. What %?
 $\frac{110}{600} = \frac{11}{60}$ for the third road. What %?

Reductions and conversions.

8. Reduce 24 ft. to meters. ($24 \div 3.28 = ?$)
9. Change 9 oz. to the decimal of a lb. ($9 \div 16 = ?$)
10. Change 36 ft.-lb to ft.-tons. ($36 \div 2,000 = ?$)
11. Convert 8 cubic ft. of water to lb. per sq. in. ($8 \times .4333 = ?$)
12. Reduce 25 miles per hr. to knots. ($25 \times .8684 = ?$)
13. Change 12 H. P. hours to kilowatt-hours. ($12 \times 1.34 = ?$)
14. Find the value of $215 \times \sqrt{24.2}$.

HINT: Set the rider to 24.2 on the right-hand end of A, bring 1 of C to the rider and move the rider to 215 on C, when under it we find the answer on D.

To find the wages due.

15. If we wish to find the wages due for N hr. at \$48 per week for 44 hr., we have the proportion $\frac{44}{?} = \frac{N}{?}$

16. Find the wages due for 26 hr. if a man receives \$42 for a 48-hr. week.

17. Find the rate of interest on $2\frac{1}{4}\%$ consols at $112\frac{3}{4}$, neglecting brokerage.

HINT:
$$\frac{112\frac{3}{4} - 100}{100} = ?$$

18. An article costing \$20 is sold at \$30 less 25% . Find the per cent of gain on the cost. What is the per cent of gain on the selling price?

19. Given:

Sales.....	\$500
Cost of goods sold.....	260
Selling expenses.....	110
General expenses.....	45
Profit.....	

What per cent of the sales is each item?

20. Find the amount due an employee if he has worked $44\frac{1}{2}$ hr. at \$20 per 48 hr. wk.

21. Find the selling price of an article bought for \$4 on which a 24% profit (on cost) is to be made.

22. If an article costs \$2.35 and is to be sold so as to make 20% on the selling price find the selling price.

HINT: Cost will be 80% of selling price, so set .8 of C scale to 2.25 of D scale and under 1 of C scale read the answer.

23. Work the following with the slide rule:

COST	TO MAKE ON SELLING PRICE	SELLING PRICE
(a) \$ 3.00	22%	
(b) 5.00	34%	
(c) 12.25	18%	
(d) 9.00	40%	
(e) 16.00	45%	

24. What amount is due an employee for $4\frac{1}{2}$ hr. of overtime work at time and a half, when the regular 44 hr. weekly wage rate is \$25?

25. What is the value in £ of \$75, when the exchange rate is 3.91?

HINT: $75 \div 3.91$

26. If exchange is 3.94, find the value in dollars of £40

HINT: Set 3.94 of C to 40 of D, and under 1 of C read the answer.

27. Find the value in francs of \$75, when exchange is 6.97.

28. Find the value of 1,000 francs if exchange is 6.91.

29. Find the value in lira of \$85 when exchange is 4.09.

30. Find the value of 2,500 lira in dollars if exchange is 3.95.

31. The list price is \$15, less 10% and 5%. Find the net cost.

32. Find the interest on a 4% Liberty Bond of \$50 from Jan. 1, 1921, to Mar. 16, 1921.

HINT:
$$\frac{.04 \times 50 \times 74}{365}$$

Rules for Characteristic.

Multiplication. If the slide projects to the left, the characteristic equals the sum of the characteristics of the factors -- if to the right, it equals the sum + 1.

Division. If the slide projects to the left, it equals the characteristic of the dividend, minus the characteristic of the divisor - 1 -- if to the right it equals the difference.

CHAPTER XVIII

DENOMINATE NUMBERS

208. Denominate Numbers. —A denominate number is a number with a specific name, such as \$5, 4 yd., 6 lb., 8 meters, etc. Sooner or later any person is apt to have occasion to use denominate numbers. Accordingly it is thought best to introduce a short chapter of these numbers in this book both as text for the student and as reference matter for the business man, who has probably studied this subject at some time in his school career and then forgotten most, if not all, of it.

There are several tables of these numbers, including both the English and the Metric systems. A few simple exercises in connection with them have been introduced, together with methods of solution, in order that the reader may recall them.

209. Tables.—

LONG MEASURE

12 in.	= 1 ft.
3 ft.	= 1 yd.
$5\frac{1}{2}$ yd. or $16\frac{1}{2}$ ft.	= 1 rd.
320 rd.	= 1 mi.
1780 yd.	= 1 mi.
5280 ft.	= 1 mi.

SURVEYORS LONG MEASURE

7.92 in.	= 1 link
25 links	= 1 rd.
4 rd. or 100 links	= 1 chain
80 chains	= 1 mi.

SAILORS MEASURE

6 ft.	= 1 fathom
120 fathoms	= 1 cable length
1.15 land mi. per hr.	= 1 knot, or 1 nautical mi. per hr., or 6080.27 ft. per hr.

SQUARE MEASURE

144 sq. in.	= 1 sq. ft.
9 sq. ft.	= 1 sq. yd.
$30\frac{1}{4}$ sq. yd.	= 1 sq. rd.
160 sq. rd.	= 1 acre
640 acres	= 1 sq. mi.

SURVEYORS SQUARE MEASURE

625 sq. links	= 1 sq. rd.
16 sq. rd.	= 1 sq. chain
10 sq. chains	= 1 acre
640 acres	= 1 sq. mi.
36 sq. mi.	= 1 township

In government surveys, 1 sq. mi. is noted as a section.

CUBIC MEASURE

1728 cu. in.	= 1 cu. ft.
27 cu. ft.	= 1 cu. yd.
128 cu. ft.	= 1 cord of wood
1 cu. yd.	= 1 load of earth
$24\frac{3}{4}$ cu. feet	= 1 perch of stone or masonry

ANGULAR MEASURE

60 sec.	= 1 min.	(60 seconds denoted 60")
60 min.	= 1 degree	(60 minutes " 60')
90 degrees	= 1 right angle	(1 degree " 1°)
360 degrees	= 1 circumference	

TROY WEIGHT (used in weighing gold, etc.)

24 gr.	= 1 pennyweight
20 pennyweights	= 1 oz.
12 oz.	= 1 lb.

AVOIRDUPOIS WEIGHT

16 oz.	= 1 lb.
100 lb.	= 1 hundredweight
2000 lb.	= 1 ton
2240 lb.	= 1 long ton (used in coal, etc., transactions—wholesale)

APOTHECARIES WEIGHT

20 gr.	= 1 scruple
3 scruples	= 1 dram
8 drams	= 1 oz.
12 oz.	= 1 lb.

COMPARATIVE WEIGHTS

1 lb. troy or apothecaries'	= 5760 gr.
1 oz. " " "	= 480 "
1 lb. avoirdupois	= 7000 "
1 oz. " "	= 437½ "

MISCELLANEOUS WEIGHTS

1 bbl. of flour	= 196 lb.
1 " " beef	= 200 "
1 cu. ft. of water weighs	62½ lb. (about 7½ gal.)
1 bu. wheat	= 60 lb.
1 bu. oats	= 32 "
1 bu. potatoes	= 60 "
1 bu. apples	= 56 "

LIQUID MEASURE

4 gills	= 1 pt.
2 pt.	= 1 qt.
4 qt.	= 1 gal. = 231 cu. in.
31½ gal.	= 1 bbl.
63 gal.	= 1 hogshead

DRY MEASURE

$$2 \text{ pt.} = 1 \text{ qt.}$$

$$8 \text{ qt.} = 1 \text{ pk.}$$

$$4 \text{ pk.} = 1 \text{ bu.} = 2150.42 \text{ cu. in.}$$

MEASURES OF TIME

$$60 \text{ sec.} = 1 \text{ min.}$$

$$12 \text{ mo.} = 1 \text{ yr.}$$

$$60 \text{ min.} = 1 \text{ hr.}$$

$$360 \text{ da.} = 1 \text{ commercial yr.}$$

$$24 \text{ hr.} = 1 \text{ da.}$$

$$365 \text{ da.} = 1 \text{ common yr.}$$

$$7 \text{ da.} = 1 \text{ wk.}$$

$$366 \text{ da.} = 1 \text{ leap yr.}$$

$$30 \text{ da.} = 1 \text{ commercial mo.}$$

$$100 \text{ yr.} = 1 \text{ century}$$

Centennial years divisible by 400, and other years divisible by 4 are leap years.

MEASURES OF VALUE

UNITED STATES MONEY

$$10 \text{ mills} = 1 \text{ cent}$$

$$10 \text{ cents} = 1 \text{ dime}$$

$$10 \text{ dimes} = 1 \text{ dollar}$$

$$10 \text{ dollars} = 1 \text{ eagle}$$

$$1 \text{ far.} = \frac{50}{99} \text{ cents; } 1 \text{ d} = 2\frac{2}{99} \text{ cents; } 1 \text{ s} = 24\frac{1}{3} \text{ cents.}$$

ENGLISH MONEY

$$4 \text{ farthings} = 1 \text{ penny (d)}$$

$$12 \text{ pence} = 1 \text{ shilling (s)}$$

$$20 \text{ shillings} = 1 \text{ pound sterling}$$

$$= \$4.8665$$

FRENCH MONEY

$$100 \text{ centimes} = 1 \text{ franc} = \$.193$$

GERMAN MONEY

$$100 \text{ pfennigs} = 1 \text{ mark} = \$.238$$

MISCELLANEOUS MEASURES

$$12 \text{ things} = 1 \text{ doz.}$$

$$12 \text{ doz.} = 1 \text{ gross}$$

$$12 \text{ gross} = 1 \text{ great gross}$$

$$24 \text{ sheets} = 1 \text{ quire}$$

$$20 \text{ quires} = 1 \text{ ream}$$

$$= 480 \text{ sheets}$$

210. Reducing to Lower Denominations.—It is sometimes necessary to reduce a given quantity to a lower denomination, or to reduce quantities of different denominations to the same denomination.

Illustrative Example 1. How many gills in 5 gal. 3 qt. 1 pt.?

SOLUTION:

$$\begin{aligned} 5 \text{ gal.} &= 20 \text{ qt.} \\ 20 \text{ qt.} + 3 \text{ qt.} &= 23 \text{ qt.} \\ 23 \text{ qt.} &= 46 \text{ pt.} \\ 46 \text{ pt.} + 1 \text{ pt.} &= 47 \text{ pt.} \\ 47 \text{ pt.} &= 188 \text{ gills} \end{aligned}$$

Illustrative Example 2. Reduce .626 mi. to lower denominations

SOLUTION:

$$\begin{aligned} 1 \text{ mi.} &= 320 \text{ rd.} \\ .626 \text{ mi.} &= .626 \times 320 = 200.32 \text{ rd.} \\ 1 \text{ rd.} &= 5.5 \text{ yd.} \\ .32 \text{ rd.} &= .32 \times 5.5 = 1.76 \text{ yd.} \\ 1 \text{ yd.} &= 3 \text{ ft.} \\ .76 \text{ yd.} &= .76 \times 3 = 2.28 \text{ ft.} \\ 1 \text{ ft.} &= 12 \text{ in.} \\ .28 \text{ ft.} &= .28 \times 12 = 3.36 \text{ in.} \end{aligned}$$

Therefore .626 mi. = 200 rd. 1 yd. 2 ft. 3.36 in.

WRITTEN EXERCISES

Reduce:

- | | |
|-----------------------------|--|
| 1. $\frac{7}{8}$ mi. to rd. | 4. .56 bu. to qt. |
| 2. .75 gal. to pt. | 5. .564 degrees to min. and sec. |
| 3. .874 mi. | 6. .374 chains to lower denominations. |

211. Reducing to Higher Denominations.—It is sometimes necessary or convenient to change a given quantity to a higher denomination.

Illustrative Example 1. Change 1,268 hr. to higher denominations.

SOLUTION:

$$\begin{aligned} 24 \text{ hr.} &= 1 \text{ da.} \\ 1,268 \text{ hr.} &= 1,268 \div 24 = 52 \text{ da., and } 20 \text{ hr. remaining} \\ 7 \text{ da.} &= 1 \text{ wk.} \\ 52 \text{ da.} &= 52 \div 7 = 7 \text{ wk. and } 3 \text{ da. remaining} \\ 4 \text{ wk.} &= 1 \text{ mo.} \\ 7 \text{ wk.} &= 7 \div 4 = 1 \text{ mo. and } 3 \text{ wk. remaining} \end{aligned}$$

Therefore 1,268 hr. = 1 mo. 3 wk. 3 da. 20 hr.

Illustrative Example 2. Reduce 2 qt. 1 pt. to the decimal part of a gal.

SOLUTION: 1 pt. = $1 \div 2 = .5$ qt.

2.5 qt. = $2.5 \div 4 = .625$ gal.

Therefore 2 qt. 1 pt. = .625 gal.

WRITTEN EXERCISES

Reduce to higher denominations.

1. 128 pt. (liquid).
2. $3\frac{1}{4}$ qt. to decimal part of a bu.
3. 8 oz. to decimal part of a lb. troy.
4. $1\frac{1}{2}$ ft. to the decimal part of a rd.
5. 75 ft. to the decimal part of a mi.
6. 59 min. to the decimal part of a wk.

212. How to Add Denominate Numbers.—Denominate numbers may be added as follows:

Illustrative Example: Add

5 gal.	3 qt.	1 pt.
6 "	2 "	1 "
7 "	1 "	1 "

SOLUTION:

18 gal.	6 qt.	3 pt.
19 "	3 "	1 "

WRITTEN EXERCISES

1. Add 5 ft. 8 in.

6 " 6 "

7 " 7 "

2. Add 6 sq. rd. 4 sq. yd. 3 sq. ft. 12 sq. in.

8 " " 6 " " 6 " " 9 " "

3. Find the length of wire necessary to wire a rectangular field 8 rd. 5 yd. 1 ft. 9 in. by 6 rd. 4 yd. 2 ft. 8 in. with four strands of wire.

213. How to Subtract Denominate Numbers.—Denominate numbers may be subtracted as follows:

Illustrative Example. Subtract 5 ft. 10 in. from 12 ft. 9 in.

$$\begin{array}{r} \text{SOLUTION: } 12 \text{ ft. } 9 \text{ in.} = 11 \text{ ft. } 21 \text{ in.} \\ \phantom{\text{SOLUTION: }} \quad \quad \quad 5 \text{ " } \quad 10 \text{ "} \\ \hline \phantom{\text{SOLUTION: }} \quad \quad \quad 6 \text{ ft. } 11 \text{ in.} \end{array}$$

WRITTEN EXERCISES

Subtract the following:

1. From 14 lb. 8 oz.

7 lb. 7 oz.

2. From 36 rd. 4 yd. 2 ft. 8 in.

26 rd. 2 yd. 1 ft. 9 in.

3. A man sold three lots each containing $8\frac{1}{3}$ sq. rd. from a field containing $2\frac{1}{3}$ acres. How much had he left?

214. Multiplication Using One Denominate Number.—Denominate numbers may be multiplied as in the following:

Illustrative Example. Multiply 6 gal. 3 qt. 1 pt. by 6

SOLUTION:

$$\begin{array}{r} 6 \text{ gal. } \quad 3 \text{ qt. } \quad 1 \text{ pt.} \\ \phantom{6 \text{ gal. }} \quad \quad \quad 6 \\ \hline 36 \text{ gal. } \quad 18 \text{ qt. } \quad 6 \text{ pt.} \\ 41 \text{ " } \quad \quad 1 \text{ "} \end{array}$$

WRITTEN EXERCISES

Multiply the following:

1. 4 yd. 2 ft. 8 in. by 7.

2. 12 lb. 4 oz. by 15.

3. What is the weight of $6\frac{1}{4}$ cu. ft. of cast iron if cast iron is $7\frac{1}{2}$ times as heavy as water and water weighs 62.5 lb. to the cu. ft.?

215. Division Using One Denominate Number.—Denominate numbers may be divided as in the following:

Illustrative Example 1. Divide 356 gills by 4.

SOLUTION: $356 \text{ gills} \div 4 = 89 \text{ gills}$
 $89 \text{ gills} = 22 \text{ pt. } 1 \text{ gill}$
 $22 \text{ pt.} = 11 \text{ qt.}$
 $356 \text{ gills} \div 4 = 11 \text{ qt. } 1 \text{ gill}$

Illustrative Example 2. Divide 46 yd. 2 ft. 8 in. by 12.

SOLUTION: Reduce to inches and then proceed as in Example 1.

WRITTEN EXERCISES

Divide the following:

1. 37 yd. 2 ft. 8 in. by 8.
2. 16 lb. 7 oz. by 6.
3. If a man can walk 16 mi. in 6 hr., what is his rate of travel?
4. If an automobile makes 238 mi. in 8 hr., what is the rate per hr.?

216. The Metric System.—This is the system of weights and measures in use in France. It is also used quite extensively in the United States and other countries at the present time. Its great advantage is the fact that all the tables use a scale of 10.

217. Terms.—The **meter** is the unit of length and is approximately 39.37 in.

The **liter** is the unit of capacity and is equal in volume to 1 cu. decimeter.

The **gram** is the unit of weight, and is the weight of 1 cu. centimeter of distilled water in a vacuum, at its greatest density (39.2° Fahrenheit. It weighs $15.432 +$ grains, English measure.

218. Prefixes.—The three Latin prefixes denote parts of the unit:

milli- means one one-thousandth
 centi- “ “ one-hundredth
 deci- “ “ one-tenth

The four Greek prefixes denote multiples of the unit:

deka- means ten
 hecto- “ one hundred
 kilo- “ one thousand
 myria- “ ten thousand

219. Tables.—

LINEAR MEASURE

(The unit is the meter)

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hectometer (Hm.)
10 hectometers	= 1 kilometer (Km.)
10 kilometers	= 1 myriameter (Mm.)

WRITTEN EXERCISES

1. Change 356 m. to Dm.; to dm.; to Km.; to mm.
2. Reduce 2642 cm. to m.; to Dm.; to Km.
3. A rectangle is 352.6 cm. long. How many meters long is it?
4. Change 5 Km. 3 Hm. 2 Dm. 4 cm. to m.
5. Reduce .25 m. to Mm.

SQUARE MEASURE

100 sq. mm.	= 1 sq. cm.
100 sq. cm.	= 1 sq. dm.
100 sq. dm.	= 1 sq. m.
100 sq. m.	= 1 sq. Dm.
100 sq. Dm.	= 1 sq. Hm.
100 sq. Hm.	= 1 sq. Km.

LAND MEASURE

(The unit is the are)

100 centares (ca.) = 1 are (a) or 100 sq. m. (about 33 ft. square).

100 ares = 1 hectare (Ha.) or 10,000 sq. m. (about $2\frac{1}{2}$ acres).

WRITTEN EXERCISES

1. Reduce 565 sq. m. to sq. Hm.
2. Reduce .0674 sq. Km. to sq. m.
3. Reduce 1 sq. m. to sq. in. (correct to three decimals). (See comparative table below).
4. Reduce 1 sq. ft. to sq. m. (correct to three decimals).

CUBIC MEASURE

1,000 cu. mm. = 1 cu. cm.

1,000 cu. cm. = 1 cu. dm.

1,000 cu. dm. = 1 cu. m. or stere.

etc.

MEASURE OF CAPACITY

(The unit is the liter)

10 milliliters (ml.) = 1 centiliter (cl.)

10 cl. = 1 deciliter (dl.)

10 dl. = 1 liter (l)

etc.

MEASURE OF WEIGHT

(The unit is the gram)

10 milligrams (mg.) = 1 centigram (cg.)

10 cg. = 1 decigram (dg.)

10 dg. = 1 gram (g.)

10 g. = 1 decagram (Dg.)

etc.

COMPARATIVE TABLE OF METRIC VALUES *vs.* ENGLISH VALUES

1 in.	= 2.54	cm.
1 ft.	= .3048	of 1 m.
1 yd.	= .9144	of 1 m.
1 rd.	= 5.029	m.
1 mi.	= 1.6093	Km.
1 sq. in.	= 6.452	sq. cm.

1 sq. ft.	=	.0929	sq. m.
1 sq. yd.	=	.8361	sq. m.
1 sq. rd.	=	25.293	sq. m.
1 sq. mi.	=	2.59	sq. Km.
1 cu. in.	=	18.387	cu. cm.
1 cu. ft.	=	28.317	cu. dm.
1 cu. yd.	=	.7646	cu. m.
1 liquid qt.	=	.9463	l.
1 dry qt.	=	1.101	l.
1 pk.	=	8.809	l.
1 bu.	=	.3524	H.
1 gr.	=	.0648	g.
1 oz. (troy)	=	31.103	g.
1 oz. (avoirdupois)	=	28.35	g.
1 lb. (troy)	=	.3732	Kg.
1 lb. (avoirdupois)	=	.4536	Kg.
1 cu. dm. of water	=	1	l. of water and weighs 1 Kg. or 2.2046 lb.
1 cm.	=	.3937	in.
1 m.	=	39.37	in.
1 Km.	=	.6214	mi.
1 sq. m.	=	1.196	sq. yd.
1 cu. m.	=	1.308	cu. yd.
1 l.	=	1.0567	liquid qt.
1 l.	=	.908	dry qt.
1 g.	=	15.432	gr.
	=	.03215	oz. troy
	=	.03527	oz. avoirdupois
1 Kg.	=	2.2046	lb. avoirdupois
1 metric ton	=	2,204.6	lb. avoirdupois

WRITTEN EXERCISES

1. Reduce 25.55 Kg. to lb. avoirdupois.
2. Reduce 2 ft. 5 in. to m.
3. Change 60 sq. m. to sq. ft.
4. How many in. in 30 mm.?
5. Reduce 2 gal. 3 qt. 1 pt. to l.
6. Change 8,678 Kg. to tons and lower denominations.
7. If cast-iron weighs 7.113 g. per cu. cm., how many lb. does a cu. ft. weigh?
8. Find the cost of 25 yd. of cloth at \$1.26 per m.
9. How many Km. in 25 mi. (to the nearest thousandth)?

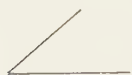
10. What is the time of traveling $\frac{1}{4}$ mi. at the rate of 100 m. in 16 sec.?
11. If a stream of water 5 ft. wide and 9 in. deep is flowing at the rate of 1 yd. per sec., find the weight of water in metric tons, supplied in 12 hr., if a cu. ft. of water weighs 1,000 oz.
12. Find the weight in lb. and in Kg. of 31.17 gal. of the best alcohol, specific gravity .792.
13. If the pressure of the air is about 1 Kg. per sq. cm. how many lb. is that to the sq. ft.?
14. What is the difference in yd. between 5 mi. and 8 Km.?
15. A bar of iron (specific gravity 7.8) is 6 ft. by 3 in. by 4 in. Find its weight in Kg.

CHAPTER XIX

PRACTICAL MEASUREMENTS

220. Practical Measurements.—Such measurements are what the term signifies; i.e., measurements which are of practical use to any person or any business at any time. These include the measurements of or appertaining to different kinds of angles; surfaces; polygons, including the parallelogram, the rectangle, the square, and the triangle; circles, including the diameter, the radius, the circumference, and the area; problems involving square root; area of irregular-shaped figures; solids, such as the cube, the cylinder, the cone, the prismatoid, and the sphere.

221. The Angle.—An angle is the amount of opening between two straight lines which meet at a point.

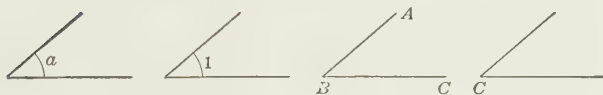


The **sides** of the angle are the lines whose intersection forms the angle. The **vertex** of an angle is the point in which the sides intersect.

222. Reading an Angle.—

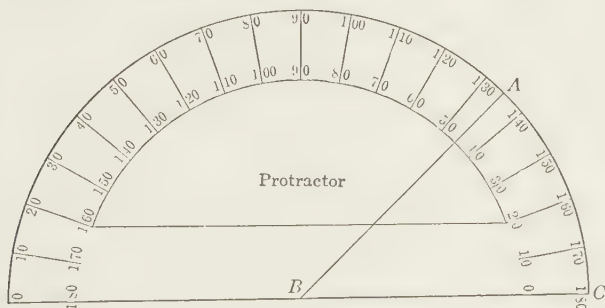
1. The best way to read an angle is to place a small letter or figure like **a** or **1** as in the following figures, and call it angle **a**, or angle **1**.
2. Another way is to use three letters, as angle **ABC** in the following figure, putting the vertex letter in the middle.

3. Another plan is to use a capital letter, as angle C in the following figure.



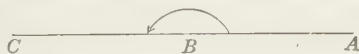
223. Unit Angle.—The unit of the angle is the degree. If we divide the circle into 360 parts, and the ends of one of these parts are joined with the center by two straight lines, the angle formed at the center is 1° .

224. The Protractor.—A protractor is a convenient instrument for measuring angles. It is a half circle with its



rim divided into 180 equal parts, called degrees of the arc. The center is also denoted at B .

To measure an angle ABC , place the protractor over the angle so that the center of the protractor is directly over the vertex of the angle and the zero mark on the scale is over one side of the angle as CB . The point where the other side, AB , of the angle ABC crosses the scale indicates the number of degrees in the angle, as 45° in this illustration.

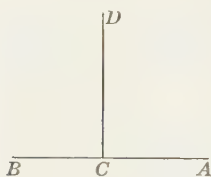


225. The Straight Angle.—This is an angle whose sides lie in the same straight line and extend in opposite directions from the vertex; as the angle ABC , in the accompanying figure.

226. The Right Angle.—This is one of two equal angles made by one straight line meeting another straight line.

Thus if the line CD meets the line AB so as to make the angle DCA equal to the angle DCB , each of these angles is a right angle.

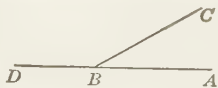
What part of a straight angle is a right angle? How many degrees in a right angle?



227. Perpendicular Lines.—A line is said to be **perpendicular** to another line when it meets it so as to form two equal angles.

What kind of angles do the lines form?

228. The Kinds of Angles.—The acute angle is an angle less than a right angle. An **obtuse** angle is an angle greater than a right angle, but less than a straight angle. ABC is an acute angle. CBD is an obtuse angle.



229. Surfaces.—A **surface** is that which has length and breadth but no thickness.

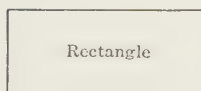
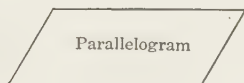
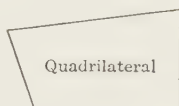
A **plane** surface is a level surface such as the surface of still water. A straight edge will fit on it in any position.

A **plane figure** is a figure all of whose points lie in the same plane.

230. Polygons.—A **polygon** is a portion of a plane bounded by straight lines, as the following figure. The **perimeter** of a polygon is the sum of all its sides. A **diagonal** is a straight line joining two non-adjacent vertices as if, in the figure below, a line should be drawn from any one corner to an opposite corner.

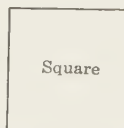
231. Quadrilaterals.—A **quadrilateral** is a plane surface bounded by **four straight lines**.

232. Parallelograms.—A **parallelogram** is a quadrilateral having its opposite sides parallel.



233. Rectangles.—A **rectangle** is a parallelogram all of whose angles are right angles.

234. The Square.—A **square** is a rectangle having four equal sides.

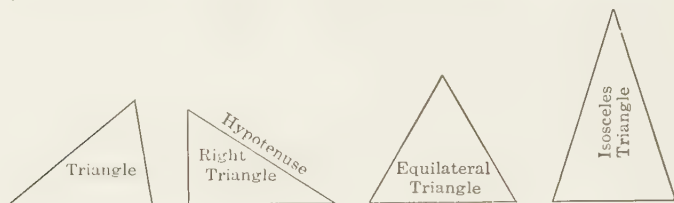


235. The Triangle.—A **triangle** is a plane figure bounded by three sides and having three angles.

236. The Right Triangle.—A **right triangle** is a triangle that has one right angle. No triangle has more than one right angle.

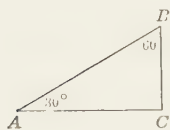
The sum of the three angles of any triangle equals two right angles or 180° .

237. The Hypotenuse.—The hypotenuse of a right triangle is the side opposite the right angle.



238. The Equilateral Triangle.—An equilateral triangle is a triangle having all its sides equal and all its angles equal.

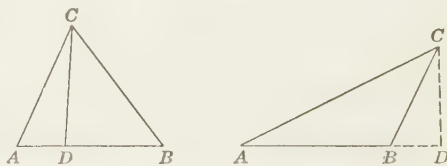
239. The Isosceles Triangle.—An isosceles triangle is a triangle having two sides equal and two angles equal.



240. A 30-60 triangle is a triangle, one of whose angles is 30° , another of whose angles is 60° and the third angle obviously 90° . The hypotenuse is twice the length of the shorter arm BC .

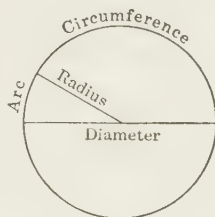
241. The base of any plane figure is the side on which it is supposed to stand, as AC in §240.

242. The altitude of any plane figure is the perpendicular distance from the opposite point highest from the base to the base or to the base extended as CD .



243. A **circle** is a plane surface bounded by a curved line, called the circumference, every point of which is equally distant from the center of the circle.

244. The **diameter** of a circle is a straight line drawn through the center and terminated by the circumference.



245. The **radius** of a circle is a straight line drawn from the center to any point on the circumference.

246. An **arc** of a circle is any part of the circumference.

247. The **perimeter** of a circle is the length of the circumference.

248. The **area** of any plane figure is the number of square units within its bounding line. A square whose side is one unit is said to have an area of one square unit. A square whose side is 1 foot is said to have an area of 1 square foot.

WRITTEN EXERCISES

1. Draw a rectangle 8 in. long and 4 in. wide, and divide it into inch squares by drawing lines parallel to the sides. Obtain the area of this rectangle by counting the number of small squares thus formed in the figure. Can you state any shorter way of obtaining the area of this rectangle?

2. Complete the following equation where $A_{\square}$ means the area of a rectangle, b = base, and a = altitude: $A_{\square} =$

3. Write the equation of Exercise 2, and then substitute the proper values. Keep all equality signs under each other and find the area of a rectangle whose length is 6 in. and whose breadth (or width) is 4 in.

4. Do the same if the length (l) = $\frac{1}{2}$ in. and the height or width, or altitude (a) is $\frac{1}{4}$ in.

5. Find the area of a rectangle whose base (b) is .25 of an in. and whose altitude (a) is .125 in.

6. A tennis court is 78 ft. long and 36 ft. wide. How many square feet does it contain? What part of an acre is it?

7. Find the perimeter and the area of a rectangle 15 yd. by 12 yd.

8. How many paving blocks 1 ft. long and 5 in. wide will be required to pave a street 2 mi. long and 35 ft. wide?

9. A rectangular field is 40 rd. long and 20 rd. wide. Find the cost of fencing it at \$2.25 a rod.

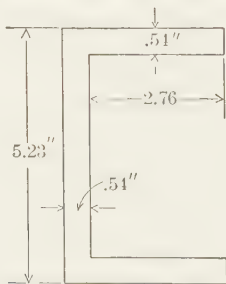
10. Find the cost of painting the four side walls of a room 12 ft. long, 10 ft. 6 in. wide, and 9 ft. high at 12 cents per sq. yd., no allowance being made for openings.

11. The length of a rectangular piece of iron is 85.24 in. and the width is 34.75 in. Find its area and perimeter.

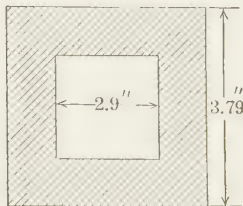
12. If 1 sq. ft. of the above mentioned iron weighs 5.1 lb. what is the weight of the entire piece if of same thickness throughout?

13. The plan of a slide valve is 10.5 in. by 7.75 in. and the pressure back of it is 85 lb. per sq. in. Find the total force pressing the valve.

14. Find the area of a channel iron from the dimensions in the accompanying figure.



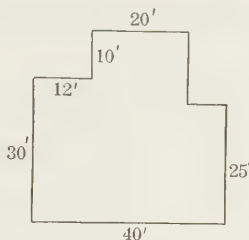
15. Find the area of the shaded part in the accompanying hollow square.



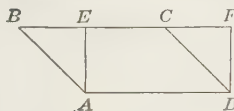
16. How many pieces of sod will it take to sod a lawn 24 ft. wide and 28 ft. long if the pieces are 12 in. by 14 in.?

17. The area of a rectangle is 180 sq. in., and its base is 4 yd. Find the altitude.

18. Find the area of a floor from the dimensions in the accompanying figure.



249. **To Find the Area of a Parallelogram.**—The parallelogram $ABCD$ may be shown equal in area to the rectangle $AEFD$ by cutting off the triangle ABE and placing it on the triangle CDF . This shows that the equation for the area of a parallelogram is then the same as that for the rectangle. What is that equation?



$$A_{\square} = ?$$

WRITTEN EXERCISES

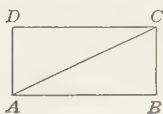
1. Find the area of a parallelogram whose base (b) is 8 in. and whose altitude (a , or AE in above figure) is 6 in.

2. Complete the following form for parallelograms whose dimensions are:

BASE	ALTITUDE	AREA OF PARALLELOGRAM
12 in.	8 in.	
6.5 in.	3.25 in.	
$9\frac{1}{2}$ ft.	$4\frac{3}{4}$ ft.	
10.2 in.	7.45 in.	

3. A piece of metal in the form of a parallelogram has an area of 127.89 sq. in. and the base is 6.3 in. Find the altitude.

250. To Find the Area of a Triangle.—If we draw the diagonal AC in the rectangle $ABCD$, then cut through the diagonal, we shall find that the triangle ABC will exactly fit on the triangle ACD . A triangle may therefore be shown to be equal in area to one-half of the area of a rectangle with the same base and the same altitude.

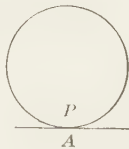


State then the equation for the area of a triangle whose base is b and whose altitude is a . Call the left member of the equation A_{Δ} .

WRITTEN EXERCISES

1. Find the area of a triangle whose base (b) is 12 in. and whose altitude (a) is 8 in.
2. What is the area of a triangle whose base is 8.5 ft. and whose altitude is 4.5 ft.?
3. Find the altitude of a triangle whose area (A) is 144 sq. in. and whose base (b) is 48 in.

251. To Find the Circumference of a Circle.—Find the length of the circumference of a circle by taking a cardboard circle, marking some point on it, as P , where circle touches level as A and

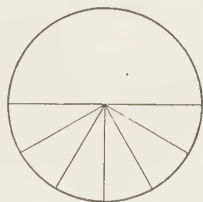


roll it along on a level surface until P again touches the level surface—say B . The distance AB will then represent the circumference of this circle. It will be found also that this length divided by the diameter of this circle will give approximately 3.1416, called π (pi). Therefore, in the accompanying figure $C \div D = \pi$, or $C = \pi D$.

WRITTEN EXERCISES

1. Find the circumference of a circle whose diameter is 8 ft.
2. A circle is 12.3 in. in diameter. What is its circumference?
3. State another name for the circumference of a circle.
4. A bicyclist travels 880 ft. per min. The bicycle wheels are 28 in. in diameter. Find how many times each wheel revolves per min.
5. A flywheel 10.5 ft. in diameter revolves 150 times in 1 min. Find the distance that a point on the rim travels in 1 min.
6. The wire from a signal tower to the signal is 450 yd. long, and the guide pulleys on the posts are $1\frac{1}{2}$ in. in diameter. Assuming that the wire must be pulled 12 in. to cause the signal to drop, how many revolutions must a pulley make?

252. To Find the Area of a Circle.—Imagine the circle divided up into many small parts as shown in the figure. It will be observed that we have practically numerous small triangles whose altitude is the radius of the circle and whose base is an arc of the circle. We then note that the sum of these arcs is the circumference of the circle. We then have the area of the circle,



$$A_{\bigcirc} = \frac{C \times r}{2}, \text{ but we have already seen that}$$

$$C = \pi D, \text{ or } C = 2 \pi r, (r = \text{radius}).$$

$$\therefore A_{\bigcirc} = \pi r \times r$$

$$A_{\bigcirc} = \pi r^2 \text{ or } A_{\bigcirc} = \pi \text{ times the square of the radius.}$$

WRITTEN EXERCISES

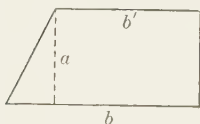
1. Find the area of a circle whose radius is 4 in.
2. Find the area of a circle whose radius is 8 in. $\pi = 3.1416$ or $\frac{22}{7}$
3. Compare the results of Exercises 1 and 2.
4. The radius of a high pressure cylinder of a marine engine is 12 in., and the effective steam pressure at a certain instant is 45 lb. per sq. in. Find the force working down on this piston if such force is equal to the product of the area of the cross section of the cylinder in sq. in. by the effective pressure in lb. per sq. in.

5. The diameter of a lever safety valve is 3 in., and the steam blows off at 95 lb. per sq. in. Determine the upward pressure on the valve.

6. An Egyptian obtained the area of a circle by subtracting from the diameter one ninth of its length and squaring the remainder. Try this plan on a circle whose diameter is 9 in., and then solve it by the above method and obtain the amount of difference of areas.

7. The boiler of an engine has 300 tubes, each 3 in. in diameter, for conducting the heat through the water. Find the total cross sectional area.

8. A piece of land is circular and 20 ft. in diameter. A circular walk 5 ft. wide is laid around it. What is the cost of this walk at \$1.75 per sq. ft.



253. The Trapezoid and its Area.—The trapezoid is a quadrilateral having only two sides parallel. The area of a trapezoid is the product of one-half of the altitude by the sum of its bases b and b' .

$$A_{\text{trapezoid}} = \frac{a}{2}(b + b')$$

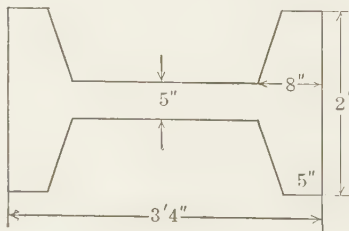
WRITTEN EXERCISES

1. Find the area of a trapezoid whose bases are 12 in. and 10 in. and whose altitude is 4 in.

2. The bases of a trapezoid are 4 ft. 6 in. and 2 ft. 8 in., and the altitude is 1 ft. 8 in. Find the area in sq. in.

3. Find the area of the accompanying figure.

4. The area of a trapezoid is 66, $b = 14$, $b' = 8$. Find the altitude.



254. Extracting Square Root.—The square of a number is the result obtained by multiplying some number by itself, as $5^2 = 5 \times 5 = 25$, and we say that the square of 5 is 25.

The square root of a number is one of the two equal factors of that number. From the statement above it is obvious that the square root of 25 is 5. This may be represented by the following ways. $\sqrt{25}$ or $25^{\frac{1}{2}} = 5$.

WRITTEN EXERCISE

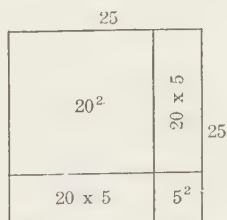
1. Complete the following form:

NUMBER	ITS SQUARE ROOT
9	
16	
36	
49	
64	
125	
1000	
10000	

It will be observed from the above form that the square root of any number between 1 and 100 is between 1 and 10; of a number between 100 and 10000 is between 10 and 100.

The square of 25 may be found as follows, or as shown in the accompanying figure.

$$\begin{array}{r}
 20 + 5 \\
 20 + 5 \\
 \hline
 (20 \times 5) + 5^2 \\
 20^2 + (20 \times 5) \\
 \hline
 20^2 + 2(20 \times 5) + 5^2
 \end{array}$$



This may be stated as follows:

The square of any number of two figures is equal to the square of the tens plus twice the product of the tens by the units plus the square of the units.

By applying this principle the square root of any number may be obtained.

The square of any number will contain twice as many figures or one less than twice as many figures as the number. Therefore separate the number into groups of two figures each beginning at the decimal point and working each way from it. There will be as many figures in the square root as there are groups in the number.

Illustrative Example. Find the square root of 625, or $\sqrt{625} = ?$

SOLUTION: Begin at the decimal point and separate the number into groups of two figures each. The largest square in 6 is 4, and the square root of 4 is 2. Obtain the remainder 2 and annex the next group (25), giving 225.

$$\begin{array}{r} 25 \\ \overline{6)25} \\ 4 \\ \hline 40 25 \\ 5 25 \\ \hline 45 \end{array}$$

Having taken the square of the tens from the number, therefore the remainder (225) must contain twice the product of the tens by the units plus the square of the units. Two times 2 tens or 20 = 40. 40 is contained 5 times in 225. 5 then is the units figure of our root. Two times the tens, times the units, plus the square of the units, is the same as the sum of two times the tens, and the units, times the units. Hence, we add 5 units to the 40 and multiply the sum by 5, obtaining 225. Therefore the square root of 625 is 25.

PRINCIPLE: To obtain the square root of a number:

1. Beginning at the decimal point, separate the number into groups of two figures each.
2. Take the square root of the greatest perfect square contained in the left-hand group for the first root figure; subtract its square from the left-hand group, and to the remainder bring down the next group.
3. Divide the number thus obtained, exclusive of its units, by twice the root figure already found for a second root figure; place this figure at the right of the root figure already found and also add this figure to the trial divisor just used. Multiply this sum by the last root figure. Subtract and proceed in a similar manner until the root is obtained.

HINTS:

1. If the divisor is greater than the remainder, place a zero in the root and also at the right of the divisor, bring down another group and proceed as before.
2. If the root of a mixed decimal is required, form groups each way from the decimal point. The last group at the right must have two figures, even though a zero must be annexed to form it.
3. To find the root of a common fraction first reduce the fraction to a decimal, then obtain the root of that decimal.
4. Obtain all roots to three places (at least) of decimals for accuracy.

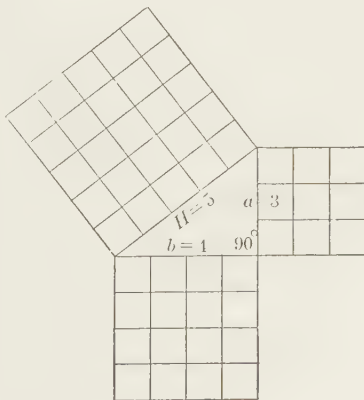
WRITTEN EXERCISES

Find the square root of each of the following:

- | | | | |
|-----------|------------|------------|--------------------|
| 1. 225. | 5. 125.44 | 9. .000624 | 13. $\frac{3}{8}$ |
| 2. 576. | 6. 50.2681 | 10. 482. | 14. $\frac{7}{8}$ |
| 3. 1225. | 7. .008 | 11. 25.8 | 15. $\frac{3}{4}$ |
| 4. 42436. | 8. 2. | 12. 3. | 16. $\frac{9}{16}$ |

255. Proportions of the Right Triangle.—It is shown in the figure and it is proved in geometry that the square on the hypotenuse of a right triangle is equal to the sum of the squares formed on each of the legs of the right triangle.

Therefore the hypotenuse equals the square root of the sums of the squares of



the two legs of the right triangle, or stated as an equation

$$H = \sqrt{a^2 + b^2}$$

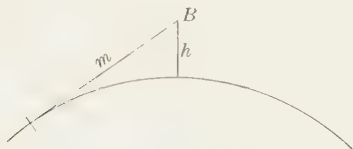
It may also be proved that $a = \sqrt{H^2 - b^2}$ or that $b = \sqrt{H^2 - a^2}$.

Illustrative Example. Find H if $a = 12$, $b = 16$.

$$\begin{aligned} \text{SOLUTION: } H &= \sqrt{a^2 + b^2} \\ &= \sqrt{12^2 + 16^2} \\ &= \sqrt{144 + 256} \\ &= \sqrt{400} \\ &= 20. \end{aligned}$$

WRITTEN EXERCISES

1. Find H , and the area of a right triangle if $a = 20$ in., $b = 25$ in.
2. The distance from home to first base on a baseball diamond is 90 ft., and the distance from first to second base is 90 ft., find the distance from second base to home in a straight line.
3. If a park is rectangular in shape and 890 yd. long by 150 yd. wide, how much is saved by walking from one corner to the opposite corner along a diagonal walk instead of walking along the sides?

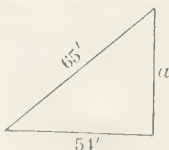


4. If a body is at B , h ft. above the surface of the earth the number of miles m , at which it can be seen, is limited by the curvature of the earth. This distance is obtained by the equation $m = \sqrt{\frac{3h}{2}}$

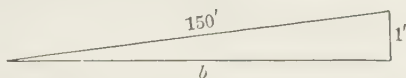
Use this in the following:

The light of a certain lighthouse is 200 ft. above the sea level. How many miles distant can it be seen?

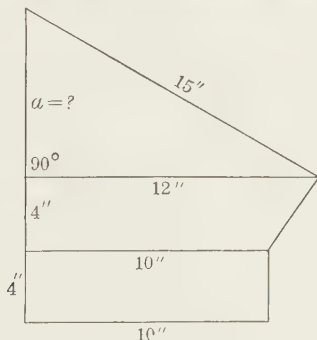
5. The back stay of a suspension bridge is 65 ft. long, and the distance of the anchoring point from the foot of one of the piers is 54 ft. Find the height (a) of the pier.



6. A railway incline is 1 ft. in 150 ft. What is the projected or horizontal length (b)?



7. Find the total area of the accompanying figure by dividing it up into parts, as indicated, then finding the areas of each and then the total.



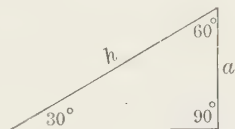
8. The hypotenuse of a right-triangle is 30 in. The altitude is 18 in.

(a) Find the base.

(b) What is the side of a square whose area is equal to area of this triangle?

(c) Find perimeter of triangle.

256. Proportions of the 30° 60° Triangle.—The hypotenuse is twice the shorter arm, a . The angle opposite the hypotenuse is 90° , therefore the principles of the preceding section apply.

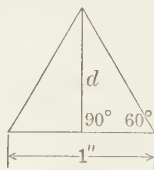


WRITTEN EXERCISES

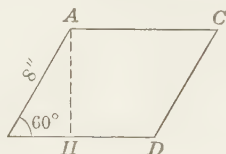
1. The side opposite the 30° angle is 8, what is the hypotenuse? Find the base by applying the principle in § 255.

2. The base of a 30° - 60° triangle is 9 in., the hypotenuse is 10.392 in. Find the other leg of the triangle, and the area of the triangle. Find the perimeter of the triangle.

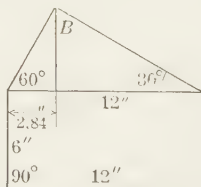
3. If the pitch of a 60° thread is 1 in., find the depth (d).



4. Find the altitude AH of the rhombus depicted here if $AC' = BD = AB = 8$ in., and $\angle ABD = 60^\circ$. Find the area.

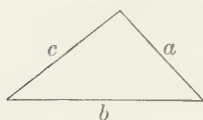


5. Find the number of square inches in the surface of the piece of sheet iron, shown in accompanying figure.



257. To Find Area of a Triangle, Given Three Sides —

The area of a triangle, given the three sides without an altitude, is sometimes required. It is obtained by taking the square root of the product of one-half the sum of the sides by one-half the sum of the sides minus one of the sides by one-half the sum of the sides minus the second side by one-half the sum of the sides minus the third side, or, expressed as a working equation,

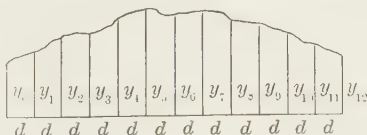


$$A_{\triangle} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \left(\frac{a+b+c}{2} \right)$$

WRITTEN EXERCISES

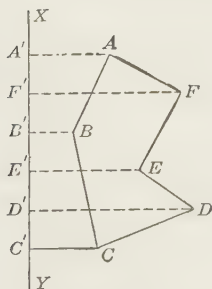
1. Find the area of a triangular piece of land whose sides are 8 rd. 12 rd. and 16 rd.
2. Find the area of a piece of tin whose sides are 4 in., 6 in., and 9 in.
3. The sides of an army camp were 7, 5, and 4 mi. What was the area of the camp in sq. mi.?
4. A piece of metal has the form of a quadrilateral. The diagonal one way is 7 in. Two of the sides on one side of this diagonal are 5 in. and 4 in. The two sides on the opposite side of this diagonal are 6 in. and 3 in. Find the area of the whole piece of metal.

258. To Find Areas of Surfaces.—The required area of the figure may be found approximately by joining the extremities of the offsets by straight lines, and then finding the sum of the areas of the trapezoids thus contained.



THE TRAPEZOIDAL RULE:

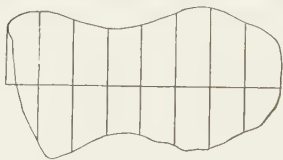
To half the sum of the first and last offsets add the sum of all intermediate offsets, and multiply this result by the common distance between them.



ANOTHER METHOD FOR FINDING THE AREA OF AN IRREGULAR PIECE: Find the distance of each vertex as A , F , etc., in the figure from a given base line as XY . These distances are called **offsets** and are the bases of trapezoids whose alti-

tudes are $A'B'$, $B'C'$, $C'D'$, etc. The area of $ABCDEF$ may now be found by the proper additions and subtractions.

SIMPSON'S RULE: Add together the first ordinate, (the perpendicular) the last ordinate, twice the sum of the other odd ordinates, and four times the sum of the even ordinates. Multiply this sum by the extreme length of the diagram and divide the result by three times the number of parts into which the diagram is divided.



WRITTEN EXERCISES

1. State Simpson's rule as an equation, using the following notation:

A = area

L = length of diagram.

y_1 = first ordinate.

y_l = last ordinate.

y_2, y_3 , etc. = the other ordinates.

n = number of parts into which the diagram is divided.

2. In the accompanying figure:

$$y_1 = \frac{1}{4}''$$

$$y_2 = 1\frac{3}{8}''$$

$$y_3 = 1\frac{1}{2}''$$

$$y_4 = 1\frac{7}{8}''$$

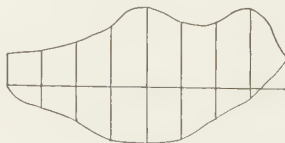
$$y_5 = 1\frac{3}{16}''$$

$$y_6 = 1\frac{1}{8}''$$

$$y_7 = 1\frac{1}{32}''$$

$$y_8 = 1\frac{3}{4}''$$

$$L = 2\frac{3}{4}''$$

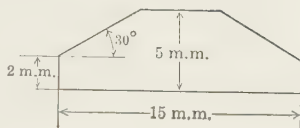


Find the area.

3. Trace the accompanying figure, measure the lines, and find the area.



4. The diagram shows the cross section of a gun metal oil ring. Find its area by the above method, then check by finding the area in some other manner.



259. Solids.—A rectangular solid is bounded by six rectangular surfaces. It is called a **prism**. The bases of a prism are parallel and equal.

PRINCIPLE: The volume of a rectangular solid is equal (in cubic units) to the product of the number of like units in its three dimensions.

WRITTEN EXERCISES

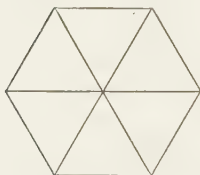
1. Find the volume of a piece of metal 6 in. by 4 in. by $\frac{1}{2}$ in.
2. What is the volume of a tank 25 ft. by 15 ft. by 8 ft.?
3. A cube has an edge of 9.54 in. Determine its volume in cu. in. and cu. ft., and its surface in sq. in. and sq. ft.
4. A cubical tank is $\frac{2}{3}$ full of water. An edge of this cube is 9 in. How many cu. in. of water are in the tank? If 1 cu. ft. of this water weighs 62.5 lb. what is the weight of the water in the tank?
5. A piece of steel of $\frac{3}{4}$ in. square section is chosen to make a lathe tool. Determine the weight, if its length is 7.25". 1 cu. in. of steel weighs .28 lb.
6. A bar of steel $2\frac{1}{2}$ in. square and 2 ft. long is molded into a square bar 12 ft. long. Find the dimensions of the bar after it is molded.
7. A rectangular tank measures on the inside $11\frac{1}{2}$ in. by $13\frac{1}{2}$ in. by 9 in. Compute the number of gallons which the tank contains when filled within an inch and a half of the top if 231 cu. in. holds 1 gal.
8. Allowing 30 cu. ft. of air per minute for each person in the classroom, how much air must be driven into the room and how many times must the air be changed during the recitation period to insure good ventilation?
9. If 38 cu. ft. of coal weigh a ton, how many tons can be put in a bin 12 ft. long, 8 ft. wide, and 6 ft. deep?
10. If 1 cu. ft. of lead weighs 700 lb., what will be the weight of a rectangular mass of lead 3 ft. 3 in. long, 2 ft. 4 in. wide, and 3 ft. high?

260. To Find the Volume of a Prism.—The volume of any prism is equal to the product of its base and its altitude.

WRITTEN EXERCISES

1. Find the volume of a triangular prism whose base is an equilateral triangle whose side is 8 in., and whose height is 12 in.

HINT: Find the altitude of the triangle, or apply § 257.

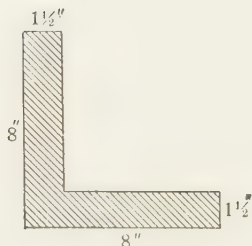
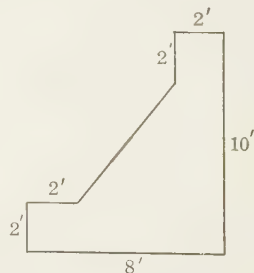


2. Find the volume of a steel rod whose base is a hexagon with side $\frac{3}{4}$ in., if the length of the rod is 10 ft.

HINT: A hexagon is composed of 6 equilateral triangles.

3. Find the volume of a steel rod whose base is the form of a rhombus with sides of 5 in., and base angle 60° , if the rod is 10 ft. long.

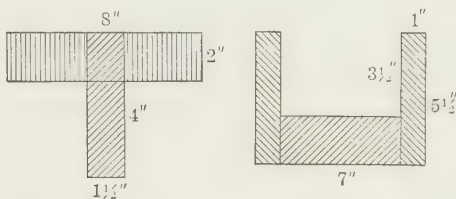
4. Find the cubic contents of a concrete retaining wall 120 ft. long, the dimensions of the cross section being as shown in the drawing.



5. In all steel construction work, such as the construction of modern buildings, the weight of the steel is computed.

In a certain building the specifications require 100 16-ft. beams, a right cross section of which is shown. Find the weight of the beams and the cost at 9¢ per lb. when put in place. (Steel weighs 490 lb. per cu. ft.)

6. In another piece of construction work, T-beams 20 ft. long and U-shaped beams 16 ft. long were used, the right sections of which are shown in the diagram. Find the weight of each.

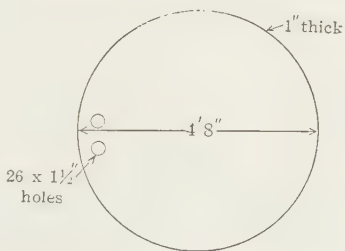


261. To Find the Volume of a Cylinder.—The volume of any cylinder is equal to the product of the base by the altitude.

WRITTEN EXERCISES

1. The diameter of the base of a right circular cylinder is 24.5 in. Find the volume if its height is 36.4 in.

2. A circular cast iron plate 1 in. thick and having 26 holes, $\frac{1}{2}$ in. in diameter is 4 ft. 8 in. in diameter. Find the weight if 1 cu. in. weighs .26 lb.



3. How high must a tomato can that is to hold 1 qt. be made, if its diameter is 4 in.?

HINT: 231 cu. in. holds 1 gal.

4. The cylinder of a steam pump, used for pumping city water, is 2 ft. in diameter and 3 ft. long. It is filled and emptied twice at each revolution of the piston. Find the number of gallons delivered by the pump in a minute, if the piston makes 24 revolutions a minute.

5. If a gasoline tank on a motor car is a cylinder 35 in. long and 15 in. in diameter, how many gallons of gasoline will it hold?

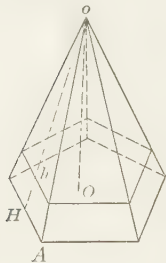
6. A hollow cylindrical can is partly filled with water. An irregular piece of iron ore is placed in the water and causes it to rise 2.5 in. in the cylinder. If the diameter of the cylinder is 8 in., what is the volume of the ore?

7. A cylinder is 20 ft. long and its volume is 1,728 cu. in. What is the diameter of the cylinder?

8. An iron pipe is a cylindrical shell 2 in. in thickness. If the pipe is 10 ft. long and its inner diameter is 12 in., and 1 cu. ft. of iron weighs 480 lb., find the weight of the pipe.

262. Pyramids.—

(a) The lateral area of any regular pyramid is equal to the product of one half of the slant height (oH) by the perimeter of the base.



(b) The lateral area of a **frustum** (lower part) of a regular pyramid is equal to one half of the sum of the perimeters of the bases multiplied by the slant height, (hH).

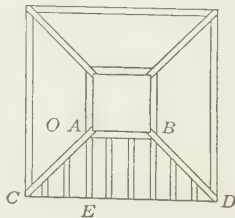
(c) The volume of any regular pyramid is equal to one third of the product of the area of the base by its altitude (oO).

WRITTEN EXERCISES

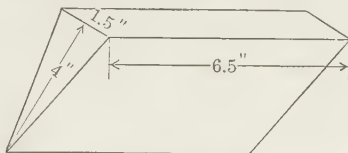
1. The slant height of a regular pyramid is 24 ft., and the base is a triangle each side of which is 8 ft. Find the lateral area.

2. The Great Pyramid of Egypt, when completed, was 481 ft. high, and each side of its square base was 764 ft. long. How many sq. ft. in the surface of the sides?

3. The figure shows the plan of a square roof in the form of a frustum of a pyramid, the upper base being a flat deck. CD is 18 ft., AB is 6 ft., and the height of the roof, or altitude AO of the frustum, is 8 ft. Find the lengths that the rafters AC and AE must be cut.



4. A wedge, whose cross-section is an isosceles triangle, base 1.5 in. and altitude 4 in., is 6.5 in. long. Find its volume. (See § 260.)



5. The area of the base of a regular pyramid is 80 sq. in. One of the lateral edges (as in fig. of § 262) is 15 in. and makes an angle of 60° with the base. Find the altitude. The volume of the pyramid.

6. Find the volume of a triangular pyramid whose altitude equals 8, and the sides of whose base are 14, 15, and 13 respectively. (See § 257.)

7. A stone spire is a hollow regular pyramid 40 ft. high, on a hexagonal base, each side of which is 10 ft. The hollow part of the spire is also a hexagonal pyramid 36 ft. high, on a hexagonal base each side of which is 9 ft. How many cu. ft. of stone in the spire?

263. The Cone.—

- (a) The lateral area of a right circular cone is equal to one-half the product of the perimeter of the base and the slant height.



- (b) The lateral area of a frustum of a right circular cone is equal to one half the product of the sum of the perimeters of the bases and the slant height.

- (c) The volume of a circular cone is equal to the area of its base multiplied by one third of the altitude.

WRITTEN EXERCISES

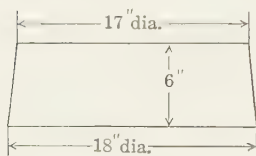
1. The slant height of a right circular cone is 8 in., and the radius of the base is 6 in. Find the lateral area. The total area.

2. How many sq. yd. of canvas will be required to make a conical tent whose altitude is 12 ft. and diameter of the base 12 ft.

HINT: Find slant height by § 255.

3. State how you would find the lateral area of the frustum of a right circular cone.

4. Find the amount of sheet metal required to make a lot of 500 pails, each 10 in. deep, 8 in. in diameter at the bottom, and 11 in. in diameter at the top, not allowing for seams nor waste in cutting.



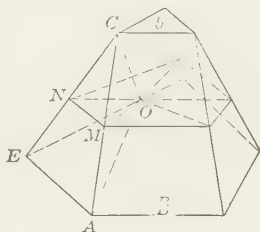
5. The accompanying diagram shows the elevation of a conical friction clutch used in automobiles. Find the contact area in sq. in., i.e. the area of the curved surface.

6. The base of a marble column is a frustum of a cone. The height is 1 ft. 6 in., and the diameters of the bases 6 ft. and 4 ft. 6 in., respectively. If 1 cu. ft. weighs 170 lb. find its weight. $\text{Vol.} = \frac{1}{3}(\text{alt.})(B + b + \sqrt{Bb})$.

7. The lateral area of a cone of revolution is 240 sq. in., and the radius of the base 6 in. Find (a) the slant height (b) the altitude (c) the volume.

264. The Prismatoid.—A prismatoid is a polyhedron bounded by two polygons in parallel planes, called the bases, and by lateral faces which are either triangles, parallelograms, or trapezoids. If one base is a rectangle and the other base a line parallel to one side of this rectangle the prismatoid is called a wedge.

The altitude is the perpendicular distance between the bases of the prismatoid.



The volume of any prismatoid is equal to the sum of its bases and four times the area of its mid-section (M), multiplied by one sixth of the altitude or $V = \frac{1}{6} a(B + b + 4M)$

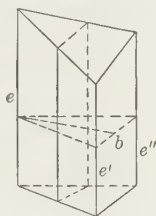
WRITTEN EXERCISES

1. Find the weight of a steel wedge whose base measures 8 in. by 5 in., the height of the wedge being 6 in., if one cu. in. of steel weighs 4.53 oz.

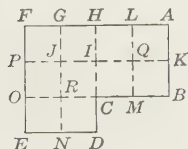
2. How much will it cost to dig a ditch 100 rd. long, 6 ft. wide at the top, $2\frac{1}{2}$ ft. wide at the bottom, and 4 ft. deep, at 60¢ per cu. yd.?

3. The volume of any truncated triangular prism is equal to one third of the sum of the three lateral edges, multiplied by the area of the right section (b).

Find the volume of a truncated triangular prism whose base is an equilateral triangle forming a right section with the lateral edges. The side of the triangle being 6 in., and the edges being 8, 10, and 6 in. respectively.



4. In order to find the contents of large excavations, the surface of the ground is laid off into small equal rectangles. Stakes are driven at the corners of all of the rectangles, such as J, I , etc., and then the surveyor finds the depth of cut to be made at each of these corners. If the rectangles are taken small their surface may be considered plane for practical purposes. The whole excavation is, therefore, divided into a number of partial volumes, each in the form of a truncated quadrangular prism. By computing the volume of each of these, and adding, we are able to obtain the whole excavation. The depth of the cut at any corner may be used 1, 2, or 3 times in the computation, depending upon the number of rectangles adjoining it.



The volume of the whole is obtained as follows:

Take each corner height as many times as there are partial areas adjoining it, add them all together, and multiply by one fourth of the area of a single rectangle.

In the figure here shown each rectangle is a square whose side is 25 ft. The depths of the cuts at the various corners are as follows: A , 2 ft.; B , 12 ft.; C , 6 ft.; D , 8 ft.; E , 8 ft.; F , 6 ft.; G , 10 ft.; H , 2 ft.; I , 6 ft.; J , 6 ft.; K , 4 ft.; L , 8 ft.; M , 6 ft.; N , 10 ft.; O , 8 ft.; P , 6 ft.; Q , 10 ft.; R , 12 ft. Find the number of cubic feet in the excavation. How many cubic yards is this?

265. The Sphere.—

(a) The area of the surface of a sphere is equal to the area of four great circles of the sphere, or $A_{\text{sphere}} = 4\pi R^2$.

(b) The volume of a sphere is equal to the product of the area of its surface by one third of the radius, or

$$V_{\text{sphere}} = \frac{4}{3} \pi R^3.$$

(c) The volume of a spherical shell (a hollow sphere) is equal to the volume of the outside sphere minus the volume of the inside sphere whose radius is r or

$$V_{\text{spherical shell}} = \frac{4}{3} \pi R^3 - \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (R^3 - r^3).$$

WRITTEN EXERCISES

1. The surface of a tiled dome, in the form of a hemispherical surface whose diameter is 24 ft. is made of colored tiles 1 in. sq. How many tiles are required to make it?

2. If a cubic foot of ivory weighs 114 lb., what is the weight of an ivory billiard ball 2 in. in diameter?

3. A hollow spherical steel shell is 1 in. thick, and its inner diameter is 8 in. How much does it weigh if there are 490 lb. to the cu. ft.?

4. If a boiler is in the form of 4 ft. cylinder 2 ft. in diameter, with hemispherical ends, how many gallons will it hold?

5. There are two spheres whose diameters are 4 in. and 8 in., respectively. Find (1) The area of each sphere. (2) The volume of each sphere. (3) The relation between the areas or the volumes of these two spheres.

6. The diameter of an arc lamp is 16 in. How many square inches of surface has the lamp, assuming it to be a sphere?

APPENDIX

TABLES AND FORMULAS

1. A Portion of a Bond Table and How It is Used.—

20 YEAR INTEREST PAYABLE SEMIANNUALLY

BONDS BEARING INTEREST AT THE RATE OF:

NET PER ANNUM	7%	6%	5%	4½%	4%	3½%	3%
4	141.03	127.36	113.68	106.84	100.00	93.16	86.32
4.10	139.32	125.76	112.20	105.42	98.64	91.86	85.09
4½	138.90	125.37	111.84	105.07	98.31	91.54	84.78
4.20	137.63	124.19	110.75	104.03	97.31	90.59	83.87
4.25	136.80	123.42	110.04	103.35	96.65	89.96	83.27
4.30	135.98	122.65	109.33	102.66	96.00	89.34	82.68
4¾	134.75	121.51	108.27	101.65	95.04	88.42	81.80
4.40	134.35	121.14	107.93	101.32	94.72	88.11	81.51
4½	132.74	119.65	106.55	100.00	93.45	86.90	80.35
4.60	131.16	118.18	105.19	98.70	92.21	85.72	79.22
4¾	130.77	117.82	104.86	98.38	91.90	85.42	78.94
4.70	129.61	116.74	103.86	97.43	90.99	84.55	78.11
4¾	128.84	116.02	103.20	96.80	90.39	83.98	77.57
4.80	128.08	115.32	102.55	96.17	89.79	83.40	77.02
4¾	126.95	114.27	101.59	95.24	88.90	82.56	76.22
4.90	126.58	113.92	101.27	94.94	88.61	82.28	75.95
5	125.10	112.55	100.00	93.72	87.45	81.17	74.90
5.10	123.65	111.20	98.76	92.53	86.31	80.09	73.86
5½	123.29	110.87	98.45	92.24	86.03	79.82	73.61
5.20	122.22	109.87	97.53	91.36	85.19	79.02	72.85
5½	121.51	109.22	96.93	90.78	84.64	78.49	72.34

Illustrative Example. 1. If I wish a 5% investment, what price can I afford to pay for a 4½% bond maturing in 20 years?

SOLUTION: Look in the left hand column for 5% and follow to the right until the column headed by 4½%. The number is 93.72, therefore I can pay as high as \$93.72.

Illustrative Example 2. If a 6% bond maturing in 20 years costs 114.27, how much will this net the buyer?

SOLUTION: Look in the 6% column and follow down to **114.27**, then follow across to the left until the left hand column is found, and you will find $4\frac{7}{8}$, therefore it will net $4\frac{7}{8}\%$.

The tables as used by the bond houses are much more extensive as to the number of years and per cents.

WRITTEN EXERCISES

1. From the above table find the rate on the investment on bonds maturing in 20 years if bought as follows:

- (a) 5% bonds bought at 103.20
- (b) 6% " " " 109.87
- (c) $3\frac{1}{2}\%$ " " " 85.42

2. Find the price at which bonds maturing in 20 years can be purchased to produce the following:

- (a) $4\frac{1}{2}\%$ bond to yield $5\frac{1}{8}\%$
- (b) 7% " " " $4\frac{1}{2}\%$
- (c) $3\frac{1}{2}\%$ " " " 5%

3. What is the rate on the investment on bonds which mature in 20 yr., if bought under the following conditions:

- (a) 7% bonds bought at 134.35
- (b) 3% " " " 73.86
- (c) 5% " " " 110.04
- (d) $4\frac{1}{2}\%$ " " " 94.94

4. Find the price at which bonds maturing in 20 yr. can be bought, to produce the following:

- (a) 5% bond to yield 4.20%
- (b) $3\frac{1}{2}\%$ " " " $5\frac{1}{8}\%$
- (c) 7% " " " $4\frac{1}{8}\%$
- (d) 6% " " " 4.9%
- (e) 5% " " " 4.1%
- (f) 4% " " " $4\frac{1}{2}\%$

2. Table Showing Powers and Roots of Some Numbers.—

No.	SQUARE	CUBE	SQUARE ROOT	CUBE ROOT	No.	SQUARE	CUBE	SQUARE ROOT	CUBE ROOT
1	1	1	1.	1.	51	2601	132651	7.1414	3.7084
2	4	8	1.4142	1.2599	52	2704	140608	7.2111	3.7325
3	9	27	1.7321	1.4422	53	2809	148877	7.2801	3.7563
4	16	64	2.	1.5874	54	2916	157464	7.3485	3.7798
5	25	125	2.2361	1.7100	55	3025	166375	7.4162	3.8030
6	36	216	2.4495	1.8171	56	3136	175616	7.4833	3.8259
7	49	343	2.6458	1.9129	57	3249	185193	7.5498	3.8485
8	64	512	2.8284	2.	58	3364	195112	7.6158	3.8769
9	81	729	3.	2.0801	59	3481	205379	7.6811	3.8930
10	100	1000	3.1623	2.1544	60	3600	216000	7.7460	3.9149
11	121	1331	3.3166	2.2240	61	3721	226981	7.8102	3.9365
12	144	1728	3.4641	2.2894	62	3844	238328	7.8740	3.9579
13	169	2197	3.6056	2.3513	63	3969	250047	7.9373	3.9791
14	196	2744	3.7417	2.4101	64	4096	262144	8.	4.
15	225	3375	3.8730	2.4662	65	4225	274625	8.0623	4.0207
16	256	4096	4.	2.5198	66	4356	287496	8.1240	4.0412
17	289	4913	4.1231	2.5713	67	4489	300763	8.1854	4.0615
18	324	5832	4.2426	2.6207	68	4624	314432	8.2462	4.0817
19	361	6859	4.3589	2.6684	69	4761	328509	8.3066	4.1016
20	400	8000	4.4721	2.7144	70	4900	343000	8.3666	4.1213
21	441	9261	4.5826	2.7589	71	5041	357911	8.4261	4.1408
22	484	10648	4.6904	2.8020	72	5184	373248	8.4853	4.1602
23	529	12167	4.7958	2.8439	73	5329	389017	8.5440	4.1793
24	576	13824	4.8990	2.8845	74	5476	405224	8.6023	4.1983
25	625	15625	5.	2.9240	75	5625	421875	8.6603	4.2172
26	676	17576	5.0990	2.9625	76	5776	438976	8.7178	4.2358
27	729	19683	5.1962	3.	77	5929	456533	8.7750	4.2543
28	784	21952	5.2915	3.0366	78	6084	474552	8.8318	4.2727
29	841	24389	5.3852	3.0723	79	6241	493039	8.8882	4.2908
30	900	27000	5.4772	3.1072	80	6400	512000	8.9443	4.3089
31	961	29791	5.5678	3.1414	81	6561	531441	9.	4.3267
32	1024	32768	5.6569	3.1748	82	6724	551368	9.0554	4.3445
33	1089	35937	5.7446	3.2075	83	6889	571787	9.1104	4.3621
34	1156	39304	5.8310	3.2396	84	7056	592704	9.1652	4.3795
35	1225	42875	5.9161	3.2711	85	7225	614125	9.2195	4.3968
36	1296	46656	6.	3.3019	86	7396	636056	9.2736	4.4140
37	1369	50653	6.0828	3.3322	87	7569	658503	9.3274	4.4310
38	1444	54872	6.1644	3.3620	88	7744	681472	9.3808	4.4480
39	1521	59319	6.2450	3.3912	89	7921	704969	9.4340	4.4647
40	1600	64000	6.3246	3.4200	90	8100	729000	9.4868	4.4814
41	1681	68921	6.4031	3.4482	91	8281	753571	9.5394	4.4979
42	1764	74088	6.4807	3.4760	92	8464	778688	9.5917	4.5144
43	1849	79507	6.5574	3.5034	93	8649	804357	9.6437	4.5307
44	1936	85184	6.6332	3.5303	94	8836	830584	9.6954	4.5468
45	2025	91125	6.7082	3.5569	95	9025	857375	9.7468	4.5629
46	2116	97336	6.7823	3.5830	96	9216	884736	9.7980	4.5789
47	2209	103823	6.8557	3.6088	97	9409	912673	9.8489	4.5947
48	2304	110592	6.9282	3.6342	98	9604	941192	9.8995	4.6104
49	2401	117649	7.	3.6593	99	9801	970299	9.9499	4.6261
50	2500	125000	7.0711	3.6840	100	10000	1000000	10.	4.6416
					π	9.8698	31.0063	1.7725	1.4646

3. Table of Decimal Equivalents of Some of the Fractions of 1 Inch.—

FRACTION OF ONE INCH	DECIMAL EQUIVALENT	FRACTION OF ONE INCH	DECIMAL EQUIVALENT
$\frac{1}{16}$	.01563	$\frac{1}{8}$	.125
$\frac{1}{32}$	.03125	$\frac{1}{4}$	.25
$\frac{1}{8}$	.0625	$\frac{1}{2}$	.5

4. Table of Wages by the Day—8 Hours to a Day.—

HOURS	\$2	\$2.25	\$2.50	\$2.75	\$3.00	\$3.25	\$3.50	\$4.00	\$4.50	\$5.00
1	.25	.28	.31	.34	.38	.41	.44	.50	.56	.63
2	.50	.56	.63	.69	.75	.81	.88	1.00	1.13	1.25
3	.75	.84	.94	1.03	1.13	1.22	1.31	1.50	1.69	1.88
4	1.00	1.13	1.25	1.38	1.50	1.63	1.75	2.00	2.25	2.50
5	1.25	1.41	1.56	1.72	1.88	2.03	2.19	2.50	2.81	3.13
6	1.50	1.69	1.88	2.06	2.25	2.44	2.63	3.00	3.38	3.75
7	1.75	1.97	2.19	2.41	2.63	2.84	3.06	3.50	3.94	4.38
8	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50	5.00

EXPLANATION: At the rate of \$3.25 per day, 5 hours' wages will be \$2.03.

Similar tables may be constructed for any commercial house at their prevailing wages.

5. Table of Formulas for Use in Commercial Work.—

$$1. \quad I = Prt.$$

P = principle; I = interest; r = rate; t = time

$$2. \quad P = \frac{I}{rt}$$

$$3. \quad r = \frac{I}{Pt (1\%)}$$

$$4. \quad t = \frac{I}{Pr}$$

$$5. \quad l = a + (n - 1) d$$

$\left\{ \begin{array}{l} l \\ a \\ n \\ d \end{array} \right.$ = last term of arith. series
 = 1st term
 = number of terms
 = common difference

6.	$S = \frac{n}{2} [2a + (n-1)d]$	S	= sum of arith. series
7.	$l = ar^{n-1}$	r	= ratio in a geom. series
8.	$S = \frac{a(r^n-1)}{r-1}$ or $\frac{rl-a}{r-1}$	S	= sum of a geom. series
9.	$A\bigcirc = \pi ab$	$\left\{ \begin{array}{l} A \\ a \\ b \end{array} \right.$	$\left\{ \begin{array}{l} = \text{area} \\ = \text{semi-major axis of an ellipse} \\ = \text{semi-minor axis of an ellipse} \end{array} \right.$
10.	$A\triangle = \sqrt{s(s-a)(s-b)(s-c)}$	$\left\{ \begin{array}{l} a, b, \text{ and } c \\ \text{are the sides of} \\ \text{triangle, } s = \frac{a+b+c}{2} \end{array} \right.$	
11.	$C\bigcirc = \pi d$	c	= circumference
12.	$A\bigcirc = \pi r^2$	d	= diameter of the circle
		r	= radius of the circle
13.	$D = \sqrt[3]{\frac{W}{.5236(A-G)}}$	$\left\{ \begin{array}{l} D \\ A \\ G \end{array} \right.$	$\left\{ \begin{array}{l} = \text{diameter of a balloon} \\ = \text{weight in lb. of 1 cu. ft. of air} \\ = \text{weight in lb. of 1 cu. ft. of gas} \end{array} \right.$
		W	= weight to be raised, including balloon
13(a).	$T = \frac{\sqrt{H}}{4} + \frac{H}{9,000}$	$\left\{ \begin{array}{l} T \\ H \end{array} \right.$	$\left\{ \begin{array}{l} = \text{number of seconds required} \\ \text{for a bomb to fall from an} \\ \text{aeroplane} \\ = \text{height in feet} \end{array} \right.$
14.	$A = P(1+r)^n$	A	= amount; P = principal; r = rate; n = no. of years
		$\left\{ \begin{array}{l} A \\ R \end{array} \right.$	$\left\{ \begin{array}{l} = \text{amount of the principal for } r \\ \text{years} \\ = \text{amount of \$1 for 1 yr. at the} \end{array} \right.$
15.	$A = \frac{S(R^n-1)}{r}$	$\left\{ \begin{array}{l} r \\ S \end{array} \right.$	$\left\{ \begin{array}{l} = \text{interest on \$1 for 1 yr.} \\ = \text{sum to be set aside annually} \end{array} \right.$
16.	$P.V. = \frac{S}{R^n} \times \frac{R^n-1}{R-1}$	$\left\{ \begin{array}{l} P.V. \\ S, R, \text{ and } n \end{array} \right.$	$\left\{ \begin{array}{l} = \text{present value of an annual} \\ \text{pension} \\ \text{as in Formula 15} \end{array} \right.$
17.	$P.V. = \frac{S}{R^b+q} \times \frac{R^q-1}{R-1}$	$\left\{ \begin{array}{l} b \\ q \\ S, R, \text{ and } P.V. \end{array} \right.$	$\left\{ \begin{array}{l} = \text{number of yr. before pension} \\ \text{begins} \\ = \text{number of yr. it is to be paid} \end{array} \right.$
		n	= no. of yr. premium should be paid in order that Life Ins. Co. shall sustain no loss
18.	$n = \frac{\log(1+\frac{Ar}{S})}{\log R}$	$\left\{ \begin{array}{l} R \\ A \end{array} \right.$	$\left\{ \begin{array}{l} = 1+r \\ = \text{amount to be paid immediately} \end{array} \right.$
		S	= amount of premium paid annually
19.	$P.V. = \frac{A}{r} (1 - \frac{1}{(1+r)^n})$	$\left\{ \begin{array}{l} P.V. \\ A \end{array} \right.$	$\left\{ \begin{array}{l} = \text{present value} \\ = \text{amount of annual pension} \end{array} \right.$
		r	= rate

20.	$R^n = \frac{Pe}{Pb}$	$\left\{ \begin{array}{l} P_e = \text{population at end} \\ P_b = \text{population at beginning} \\ R = 1 + r = \text{rate of increase of population} \end{array} \right.$
21.	$C = \frac{S[(1+r)^n - 1]}{r}$	$\left\{ \begin{array}{l} C = \text{number of dollars in debt} \\ n = \text{number of years} \\ S = \text{sum set aside annually} \\ r = \text{rate of interest} \end{array} \right.$
22.	$V = PR \frac{(R^n - 1)}{(R - 1)}$	$\left\{ \begin{array}{l} V = \text{total value} \\ P = \text{premium paid each year} \\ R = 1 + r \\ P = \text{price of a bond that has } n \text{ yr. to run} \end{array} \right.$
23.	$X = \frac{(Sq + Sr(1+q)^n - Sr)n}{Pq} - 1$	$\left\{ \begin{array}{l} r = \text{rate \% it bears} \\ S = \text{face of bond (usually \$100 or \$1,000)} \\ q = \text{current rate of interest} \\ X = \text{rate of interest yield} \end{array} \right.$
24.	$A\Box = ba$	$\left\{ \begin{array}{l} b = \text{base} \\ a = \text{altitude} \end{array} \right.$
25.	$A\Box = ba$	
26.	$A\triangle = \frac{ba}{2}$	
27.	$A \text{ (trapezoid)} = a \frac{(b+b')}{2} \text{ or } \frac{a}{2} (b+b')$	
28.	$A \text{ (irregular figure)} = \text{sum of } A\text{'s of triangles, trapezoids, etc.}$	
29.	$H = \sqrt{a^2 + b^2}$	$\left\{ \begin{array}{l} H = \text{hypotenuse of a right triangle} \\ a = \text{altitude of a right triangle when } b \text{ is base} \end{array} \right.$
30.	$m = \sqrt{\frac{3h}{2}}$	$\left\{ \begin{array}{l} h = \text{no. of ft. object is above surface of earth} \\ m = \text{no. of mi. object can be seen} \end{array} \right.$
31.	$H = 2a$	$\left\{ \begin{array}{l} H = \text{hypotenuse of a 30-60, right triangle} \\ a = \text{arm opposite the } 30^\circ \text{ angle} \end{array} \right.$
32.	$V \text{ (rectangular solid)} = abc$	$\left\{ \begin{array}{l} a = \text{1st edge, } b = \text{2d edge, } c = \text{3d edge, } V = \text{volume} \end{array} \right.$
33.	$V \text{ (pyramid)} = \frac{1}{3} Ba$	$\left\{ \begin{array}{l} B = \text{area of base and } a = \text{altitude of pyramid} \end{array} \right.$
34.	$\text{Lateral area (pyramid)} = \frac{1}{2} \text{ slant height} \times P_b$	$P_b = \text{perimeter of base}$
35.	$V \text{ (cone)} = \frac{1}{3} \text{ area of base} \times \text{altitude}$	
36.	$L. A. \text{ (cone)} = \frac{1}{2} \text{ slant height} \times \text{circumference of base}$	
37.	$V \text{ (prismatoid)} = \frac{1}{6} a (B + b + 4M)$	$\left\{ \begin{array}{l} a = \text{alt.; } B = \text{area of lower base} \\ b = \text{area of upper base} \\ M = \text{area of mid-section} \\ R = \text{radius of the sphere} \end{array} \right.$
38.	$A \text{ (sphere)} = 4\pi R^2$	
39.	$V \text{ (sphere)} = \frac{4}{3}\pi R^3$	
40.	$V \text{ (spherical shell)} = \frac{4}{3}\pi (R^3 - r^3)$	$\left\{ \begin{array}{l} R = \text{radius of outside of shell} \\ r = \text{radius of inside of shell} \end{array} \right.$

6. Abbreviations Used in Commercial Transactions.

A.....	acre	cons'g't.....	consignment
acct. or a/c.....	account	cr.....	crate; credit; credi- tor
agt.....	agent	cs.....	case; cases
amt.....	amount	csk.....	cask
ans.....	answer	cu. ft.....	cubic feet
Apr.....	April	cu. in.....	cubic inch
a/s.....	account sales	cu. yd.....	cubic yard
Aug.....	August	cwt.....	hundredweight
av.....	average	d.....	pence
bal.....	balance	da.....	day
bg.....	bag, bags	Dec.....	December
bbl. or bl.....	barrel	dep't.....	department
bdl.....	bundle; bundles	dft.....	draft
bk.....	bank; book	disc.....	discount
bl.....	bale; bales	do.....	ditto
bkt.....	basket	doz.; dz.....	dozen
B/L.....	bill of lading	Dr.....	debtor; debit; doc- tor
B/O.....	back order	E.....	East
bot.....	bought	ea.....	each
bu.....	bushel	e. g.....	for example
bx.....	box; boxes	e. o. e.....	errors and omis- sions excepted
c, c, b.....	cash; cash book	etc.....	and so forth
cash.....	cashier	ex.....	express
c, ct.....	cent	exch.....	exchange
cd.....	card; cord	exp.....	expense
car.....	carton	far.....	farthing
cg.....	centigram	Feb.....	February
ch.....	chain; chains; chest	f. o. b.....	free on board
chg.....	charge	frt.....	freight
c. i. f.....	carriage and insur- ance free	f, fr.....	franc
ck.....	check	ft.....	foot
cm.....	centimeter	gal.....	gallon
cml.....	commercial	gi.....	gill
cn.....	can	gr.....	grain
Co.....	company; county	gro.....	gross
C. O. D.....	collect on delivery	guar.....	guarantee
coll.....	collection		
com.....	commission		

hf.....	half	pdf.....	preferred
hf. cht.....	half chest	pk.....	peck; pecks
hhd.....	hogshead	pkg.....	package
hr.....	hour	pp.....	pages
i. e.....	that is	pr.....	pair; pairs
in.....	inch; inches	pt.....	pint; pints
ins.....	insurance	pwt.....	pennyweight
inst.....	instant; the present month	qr.....	quire
int.....	interest	qt.....	quart
l.; inv.....	invoice	rd.....	rod
inv't.....	inventory	rec'd.....	received
Jan.....	January	rm.....	ream
kg.....	keg; kegs	Rm. (or M.)..	Reichsmark; Mark
l.....	link; links	s.....	shilling; shillings
lb.....	pound; pounds	S.....	South
l. p.....	list price	sec.....	second
Mar.....	March	Sept.....	September
mdse.....	merchandise	set.....	settlement
Messrs.....	Gentlemen; Sirs	ship.....	shipment
mi.....	mile; miles	shipt.....	shipped
min.....	minute; minutes	sig.....	signature; signed
mo.....	month; months	sq. ch.....	square chain
Mr.....	Mister	sq. ft.....	square foot
Mrs.....	Mistress	sq. mi.....	square mile
N.....	North	sq. rd.....	square rod
no.....	number	sq. yd.....	square yard
Nov.....	November	T.....	ton
Oct.....	October	tb.....	tub
o d.....	on demand	Tp.....	township
O. K.....	correct	tr.....	transfer
oz.....	ounce; ounces	treas.....	treasurer
p.....	page	ult.....	last month
pay't.....	payment	via.....	by way of
pc.....	piece; pieces	viz.....	namely; to wit
pd.....	paid	vol.....	volume
per.....	by the; by	wk.....	week
per cent.....	per centum; by the hundred	wt.....	weight
		yd.....	yard; yards
		yr.....	year; years

7. Table of Symbols.

a/c.....	account	M	thousand
a/s.....	account sales	"	inch; inches; seconds
+	addition	>	greater than
()	aggregation	<	less than
&	and	×	multiplication
... ..	and so on	#	number if written before a figure; pounds if written after a figure
@.....	at; to	1 ¹	one and one fourth
c/o.....	care of	%	per cent
¢	cent; cents	£	pounds sterling
✓.....	check mark	∴	since
°	degree	—	subtraction
÷	division	∴	therefore
\$	dollar; dollars		
=	equal; equals		
'	foot; feet; minutes		
©	hundred		

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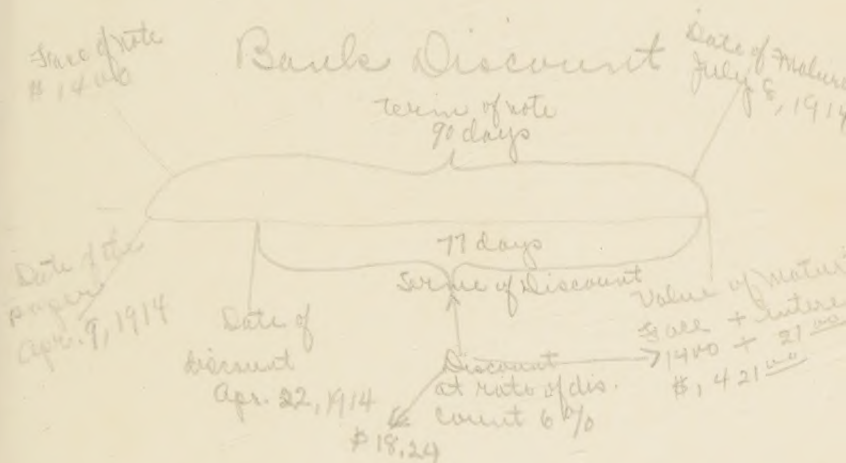
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A note dated a certain day of one month with fall due on the same day of the next mo. If there is such a day, if not on the last day of the month Jan 31 - Feb. 28 or 29.



Payee receives:

Value at maturity	\$1,421.00
less discount	- 18.24
	<u>\$ 1402.76</u>

The cost price includes the cost price +



